Math 54 Handout 2

August 29th 2016

Systems of linear equations

1. Each of the following matrices is the augmented matrix for a system of linear equations, in row echelon form. Determine in each case if the system is consistent, and if it is, determine whether the solution is unique.

$$\begin{bmatrix} \Delta & * & * & * \\ 0 & \Delta & * & * \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & \Delta & * & * & * \\ 0 & 0 & \Delta & * & * \\ 0 & 0 & 0 & \Delta & 0 \end{bmatrix} \begin{bmatrix} \Delta & * & * \\ 0 & \Delta & * \\ 0 & 0 & \Delta \end{bmatrix} \begin{bmatrix} \Delta & * & * & * & * \\ 0 & 0 & \Delta & * & * \\ 0 & 0 & 0 & \Delta & * \end{bmatrix}$$

Solution: Consistent, not unique; consistent, not unique; inconsistent; consistent, not unique.

2. Put the following matrix in reduced row echelon form.

$$\begin{bmatrix} 1 & 5 & 4 & 3 & -2 \\ 1 & 3 & 5 & 4 & 1 \\ 1 & 4 & 3 & 4 & 2 \\ 2 & 6 & 13 & 7 & -1 \end{bmatrix}$$

3. Which of these matrices are in row echelon form? Reduced row echelon form? If they are in echelon form, find the solutions.

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 4 \\ 0 & 1 & 0 & 4 \end{bmatrix} \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 3 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 & 4 \\ 0 & 1 & 0 & 5 \\ 0 & 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 4 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

Solution: With N = neither, E = echelon, and R = reduced:
N; E; E; R; R; R; R; R; N.

4. Give an example of an inconsistent underdetermined system of two equations in three unknowns.

Solution: $x + y + z = 4$ and $x + y + z = 5$; or $0 = 5$ and $x + 2y + 3z = 0$ for example.