Problem 1) Can Newton method be used to solve \( f(x) = (x - 3)^{1/2} = 0 \) given an initial approximation \( p_0 = 4 \)? Explain why.

Problem 2) Consider function \( f(x) = \sin(x) \) on the interval \([0, 1]\) on equally spaced nodes. If \( h \) is the spacing between the nodes Determine the size of \( h \) you need so that the cubic Lagrange interpolating polynomial approximates \( f(x) \) with an accuracy of \( 10^{-5} \), i.e. \( |E_3(x)| \leq 10^{-5} \) for any \( x \in [0, 1] \).

Problem 3) Let \( g(x) = 3x^4 - 8x^3 + 6x^2 \).

Part a) Show that \( p = 0 \) and \( \hat{p} = 1 \) are fixed points of the function \( g(x) \).

Part b) Assume that you used fixed point iteration with some initial approximation \( p_0 \) which was sufficiently close to \( p = 0 \) so that the sequence of approximations \( p_0, p_1, p_2, \ldots \) you generated converges to the fixed point \( p = 0 \). Derive what will be the order of convergence of this sequence.

Part c) Assume that you used fixed point iteration with some initial approximation \( \hat{p}_0 \) which was sufficiently close to \( \hat{p} = 1 \) so that the sequence of approximations \( \hat{p}_0, \hat{p}_1, \hat{p}_2, \ldots \) you generated converges to the fixed point \( \hat{p} = 1 \). Derive what will be the order of convergence of this sequence.

Problem 4) Show that any third-degree polynomial \( f(x) = a_0 + a_1 x + a_2 x^2 + a_3 x^3 \) is its own clamped cubic spline on any closed interval \([a, b]\). \textit{Hint:} use uniqueness of a clamped cubic spline interpolating a function \( f(x) \) on a given set of nodes.

Problem 5) Suppose real numbers are represented with a 5-bit mantissa and a 3-bit characteristic.

Part a) How many positive real numbers such representations system contains?

Part b) What is the magnitude of a maximum number that can be represented with this system?