POSSIBLE DRP PROJECTS

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1. Number Theory

- Dirichlet's theorem on primes in arithmetic progressions.
  Possible references: [Ser73], [Mur08], [Apo76]
- Prime Number Theorem
  Possible references: [Mur08], [Apo76]
- L-functions (analytic continuation, functional equations), possibly building up on the previous two topics
  Possible references: [Apo76], [Neu99], [Lan94], [Mur08], [Tat67]
- Fermat's sum of squares theorem. There are various different ways to prove it: By factoring in $\mathbb{Z}[i]$, using lattices and Minkowski's theorem, or using quadratic forms, character sums, etc.
  Possible references: [ME05], [Cox89], [IR82]
- Quadratic Reciprocity law. There are a lot of different ways to prove it: Prime splitting in cyclotomic fields, Gauss sums, etc.
  Possible references: [IR82], [ME05]
- Number Fields (for example with special cases of Fermat's last theorem as applications, or other diophantine equations, or the previous two projects)
  Possible references: [ME05], [Mar18]. More advanced: [CF67], [Lan94], [Neu99]
- Quadratic Forms
  Possible references: [Ser73], [Cox89], [Cas78], [Kit93], [OMe00]
- Equidistribution (aka uniform distribution) mod 1, or in more general groups. (quite analytic)
  Possible references: [KN74], [IK04, Chapter 21], [Cha68, VIII], [Fit14]
- Pisot numbers
  Possible references: [Sal63], [Ber+92]
- p-adic numbers.
  Possible references: [Kob84], [Gou20], [Rob00], [Cas86]. Also any algebraic number theory book covers local fields.
- Elliptic Curves. A possible goal might be to prove the Mordell-Weil theorem.
  Possible references: [Sil09], [Mil06]. Perhaps slightly more elementary: [Kna92], [ST15]
- Modular Forms
  Possible references: [Mil17], [Miy06], [Bum97], [DS05]

- Quaternion algebras
  Possible references: [Voi21]

1.1. **More advanced topics in Number Theory (requiring some background)**

- Class Field Theory
  Possible references: [Mil20], [Lan94], [CF67], [Neu99]
- Other topics in Algebraic Number Theory: Artin L-Functions, Hecke L-Functions, Tate's thesis, Galois Cohomology, Class Field Theory (see above)
  Possible references: References above, [CF67], [Tat67]
- L-functions of CM elliptic curves and relation to modular forms, Sato-Tate conjecture
  Possible references: [Sil94], [Iwa15], [Miy06]
- p-adic integrals, and relation to counting points mod p^n.
  Possible references: [Igu07]
- Representations of GL_2 (or more general groups) over nonarchimedean local fields (possibly with outlook towards local Langlands).
  Possible references: [Bum97], [BZ76], [BH06]

2. Geometry

- Riemann Surfaces
  Possible references: [For81], [Don11]
- De Rham cohomology
  Possible references: [Lee13], [Tu11]

3. Algebra

- Category Theory
  Possible references: [Mac98], [Rie17], [Bra17] (in German)
- Representation Theory of finite groups
  Possible references: [FH91], [Ser77]
- Lie algebras/groups and their representations
  Possible references: [FH91], [Kir08]
- Topics in Galois Theory (Kummer Theory, realizable Galois groups over $\mathbb{Q}$, computing Galois groups over $\mathbb{Q}$, etc.)
  Possible references: [Lan02], [Ser16]
- Real fields, positive rational functions as sums of squares, Artin-Schreier Theorem (on finite index subfields of algebraically closed fields)
  Possible references: [Lor07], [Lan02]

4. Analysis

- Spectral Theorem in infinite dimensions.
  Possible references: [Con85], [Rud91], [Arv02]
- Banach Algebras, C^*-algebras and related things, e.g. with applications to spectral theory
  Possible references: [Arv02]
- Fourier series
  Possible references: [SS03], [Duo01]
- Abstract Harmonic Analysis (Fourier transform for LCA groups, Peter-Weyl theorem, etc.)
  Possible references: [Fol15], [DE09]
- Distribution Theory
  Possible references: [Gru09], [Hör03]
- The Gamma function
  Possible references: [Art64], [FB09]
- Topics in Complex Analysis (Riemann Mapping Theorem, Weierstraß products, Mittag-Leffler, Picard's theorems, Elliptic functions, Modular forms (see above), etc.)
  Possible references: [Rem98], [FB09], [BWW20]
- Amenability of groups (with Banach-Tarski paradox as example)
  Possible references: [Run02], [Wag85]

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