CALCULUS REVIEW SHEET

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1. Algebra

Problem 1.1. Reduce and simplify the fractions if possible:

a) $\dfrac{x^2}{xy+x}$, b) $\dfrac{ab+c}{a}$, c) $\dfrac{a^{5n}b^{-3n}}{a^{-2n}b^4}$, d) $\dfrac{5(x+y)}{2x+2y}$,

e) $\dfrac{\frac{x^2+xy}{z^2}}{\frac{2x}{z}}$, f) $\dfrac{1-x^3}{x-1}$, g) $\dfrac{x^2+3x+2}{x+1}$, h) $\dfrac{4x^2-9y^4}{2x-3y}$.

Problem 1.2. Find all solutions $x \in \mathbb{R}$ to the equations:

a) $5x+7=1$, b) $x^2=9$, c) $x^2+2x=-1$, d) $x^3+3x^2=3x$,

e) $\dfrac{x}{x^2+2}=1$, f) $x^3-2x^2-x+2=0$, g) $x^6+x^4=-5-6x^2$, h) $\sqrt{1-2x}=x$.

2. Special Functions

Problem 2.1. What are the functions $\sin, \cos, \sec, \csc, \tan, \cot$? What are their domains? What are their derivatives and antiderivatives?

Problem 2.2. What are the inverse trig functions? What are their derivatives?

Problem 2.3. List some trig identitites you know: $\sin^2 x + \cos^2 x = ?$, $\sin(x+y) = ?$, etc.

Problem 2.4. What is the function ln? What is its derivative?

3. Limits

Problem 3.1. Recall some rules for computing limits.

Problem 3.2. Compute the following limits:

a) $\lim\limits_{n\to\infty} \dfrac{n^2}{2n^2-1}$, b) $\lim\limits_{x\to\infty} \dfrac{e^x+e^{-x}}{e^x-e^{-x}}$, c) $\lim\limits_{x\to 1} \dfrac{x^2-3x+2}{x-1}$,

d) $\lim\limits_{t\to\infty} \dfrac{e^t}{t^{500}}$, e) $\lim\limits_{x\to-\infty} \dfrac{\sqrt{1+x^2}}{x}$, f) $\lim\limits_{n\to\infty} a^n$ (where $a>0$),

g) $\lim_{x\to 0} \dfrac{\tan x}{\cos x - 1}$, h) $\lim_{n\to\infty} \sqrt{n+1} - \sqrt{n}$, i) $\lim_{x\to 0} x \sin\left(\dfrac{1}{x}\right)$.

4. Derivatives

Problem 4.1. What is the definition of the derivative of a function $f : \mathbb{R} \to \mathbb{R}$? What is its geometric interpretation?

What rules for computing derivatives do you know?

Problem 4.2. Compute the derivatives of the following functions:

a) $e^{\left((\ln x)^2\right)}$, b) $\sin(\sqrt{\cos(x^2)})$, c) $\tan(x^2+1)$, d) $\arcsin(e^{(x^2)})$.

Problem 4.3. How can you use derivatives to check

- whether a function is increasing/decreasing?
- if the function has a local extremum?

Problem 4.4. For the following functions determine their domains, intervals where the function is increasing/decreasing, local extremums:

a) $\sqrt{1-x^2}$, b) $\dfrac{1+x^2}{1-x^2}$, c) $e^x \ln x$, d) $x \ln x$.

5. Integrals

Problem 5.1. What is the relation between the area under a graph and integrals? What is an indefinite integral? What is the fundamental theorem of calculus?

Problem 5.2. Recall some integration techniques.

Problem 5.3. Compute the following integrals:

a) $\displaystyle\int_0^1 x^a \,\mathrm{d}x$ (where $a \geq 0$), b) $\displaystyle\int_1^2 x^2 + 2x \,\mathrm{d}x$, c) $\displaystyle\int \frac{\ln t}{t} \mathrm{d}t$,

d) $\displaystyle\int x e^{x^2} \mathrm{d}x$, e) $\displaystyle\int \sin(\sin(x))\cos(x) \mathrm{d}x$, f) $\displaystyle\int_{-4}^{4} \theta \cos(\theta^4) \,\mathrm{d}\theta$.

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