Math N1B Lec 002 Homework 3

8.3: 26, 28, 30,

8.5: 4 (only a, b), 6, 10, 12, 16 (for b only set up an equation to find the target weight)

11.1: 24, 30, 38, 42, 46, 80

11.2: 18, 26, 30, 34, 40, 44, 48, 52

In addition to the above problems in the book, do the following problems:

Problem 1. Use differentiation under the integral sign to compute $\int_0^1 \frac{x-1}{\ln x}\mathrm{d}x$. (Hint: Consider $\int_0^1 \frac{x^t-1}{\ln x}\mathrm{d}x$.)

Problem 2. This problem gives an elementary way of proving $\sum_{n=1}^{\infty}\frac{1}{n^2}=\frac{\pi^2}{6}$.

(i) Prove

$$\cot^2\theta < \frac{1}{\theta^2} < 1+\cot^2\theta$$

for $0<\theta<\frac{\pi}{2}$ (you may use that $0<\sin\theta<\theta<\tan\theta$ holds for these angles).

(ii) Use (i), the identity $\sum_{k=1}^{m}\cot^2\left(\frac{k\pi}{2m+1}\right)=\frac{m(2m-1)}{3}$, and the squeeze lemma to prove

$$\sum_{n=1}^{\infty}\frac{1}{n^2}=\frac{\pi^2}{6}.$$

If you don't have a copy of the book, see the bcourses assignment for pictures of the pages in the book containing the exercises.

Submit these problems on Gradescope by 07/14 23:59. You should do more practice problems in these sections for yourself.

Department of Mathematics, Evans Hall, University of California, Berkeley, CA 94720, USA

Email address: leonard.tomczak@berkeley.edu