Problem Set 10 (due November 19)
MATH 110: Linear Algebra

Each problem is worth 10 points.

PART 1

2. Curtis p. 131 12.

PART 2

Problem 1 (20)
Let $V$ be a real Euclidean space of dimension $n$. An orthogonal transformation $T : V \to V$ with determinant 1 is called a rotation. If $n$ is odd, prove that 1 is an eigenvalue for $T$.

Problem 2 (10)
If $T : V \to V$ is unitary and Hermitian, prove that $T^2 = I$.

Problem 3 (10)
If $A$ is a unitary matrix and if $I + A$ is nonsingular, prove that $(I - A)(I + A)^{-1}$ is skew-Hermitian.

Problem 4 (18)
A square matrix is called normal if $AA^* = A^*A$ (i.e. the matrix commutes with the conjugate of its transpose). Determine which of the following matrices are normal (with proof or counterexample)
   a) Hermitian matrices.
   b) Skew-Hermitian matrices.
   c) Symmetric matrices.
   d) Skew-symmetric matrices.
   e) Unitary matrices.
   f) Orthogonal matrices.

Problem 5 (10)
Show that if $A$ is a real orthogonal $2 \times 2$ matrix with determinant 1 then

$$A = \begin{pmatrix}
\cos \theta & -\sin \theta \\
\sin \theta & \cos \theta
\end{pmatrix}$$

for some $\theta$. 

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