Math 185
Summer 2019

How to write up your homework :

1. Write in complete sentences.

2. Use mathematical symbols correctly. Writing "$\longrightarrow$" has no meaning. On the other hand, $A \Rightarrow B$ means "A implies B", i.e. **if** A is true **then** B is true. Writing $A \iff B$ means "A is true if and only if B is true", i.e. that A and B are logically equivalent.

A review of logic and the writing of mathematical arguments is at https://math.berkeley.edu/~hutching/teach/proofs.pdf

3. Give complete arguments, but do not write more than is needed. Einstein said, "Make things as simple as possible, but not simpler."

4. Write legibly, with a pen or dark pencil. Do not write in a stream-of-consciousness way, by continually crossing things out.

5. Have mercy on the grader, who is not a mindreader.

p. 55 # 10
(a) From the Theorem, $\lim_{z\to\infty} f(z) = w_0$ if $\lim_{z\to 0} f\left(\frac{1}{z}\right) = w_0$. In our case, we want to take $f(z) = \frac{4z^2}{(z-1)^2}$ and $w_0 = 4$. Now

$$f\left(\frac{1}{z}\right) = \frac{4(1/z)^2}{(\frac{1}{z}-1)^2} = \frac{4}{(1-z)^2}.$$

Hence $\lim_{z\to 0} f\left(\frac{1}{z}\right) = 4$. We conclude from the theorem that

$$\lim_{z\to\infty} \frac{4z^2}{(z-1)^2} = 4.$$

(b) From the Theorem, $\lim_{z\to z_0} f(z) = \infty$ if $\lim_{z\to z_0} \frac{1}{f(z)} = 0$. In our case, we want to take $f(z) = \frac{1}{(z-1)^3}$ and $z_0 = 1$. Now

$$\frac{1}{f(z)} = (z-1)^3.$$

Hence $\lim_{z\to 1} \frac{1}{f(z)} = 0$. We conclude from the theorem that

$$\lim_{z\to 1} \frac{1}{(z-1)^3} = \infty.$$

(c) From the Theorem, $\lim_{z\to\infty} f(z) = \infty$ if $\lim_{z\to 0} \frac{1}{f\left(\frac{1}{z}\right)} = 0$. In our case, we want to take $f(z) = \frac{z^2+1}{z-1}$. Now

$$\frac{1}{f\left(\frac{1}{z}\right)} = \frac{1}{\frac{z^{-2}+1}{z^{-1}-1}} = \frac{z^{-1}-1}{z^{-2}+1} = \frac{z(1-z)}{1+z^2}.$$

Hence $\lim_{z\to 0} \frac{1}{f\left(\frac{1}{z}\right)} = 0$. We conclude from the theorem that

$$\lim_{z\to\infty} \frac{z^2+1}{z-1} = \infty.$$

p. 35 #4

(a) The set $-\pi < arg(z) < \pi, z \neq 0$, is the complex plane minus the ray extending leftwards from the origin. The closure is the complex plane.

(b) $|Rez| < |z| \iff |x| < \sqrt{x^2+y^2} \iff x^2 < x^2+y^2 \iff 0 < y^2 \iff y \neq 0$. The set is the complex plane minus the x-axis. The closure is the complex plane.

(c) $Re\left(\frac{1}{z}\right) \leq \frac{1}{2} \iff \frac{x}{x^2+y^2} \leq \frac{1}{2} \iff 2x \leq x^2+y^2 \iff 1 \leq (x-1)^2+y^2$. The set consists of the circle with radius one and center $(1,0)$ (i.e. $(x-1)^2+y^2=1$) along with everything outside of it. It is already closed.

(d) $Rez^2 > 0 \iff x^2-y^2 > 0 \iff x^2 > y^2 \iff |x| > |y|$. Thinking of the lines $y=x$ and $y=-x$ as separating the plane into four open regions, the set consists of the right and left regions. The closure comes from adding the lines $y=x$ and $y=-x$.