Tangents, Areas and Arc length

If the curve is described by parametric equations, then then tangent can be described in terms of the parameter. You should remember the formulaes for arc-length of parameterized curve and the area of the region enclosed by a parametric curve.

Excise:
Consider the astroid given by the parametric equation.

$$x = \cos^3\theta, y = \sin^3\theta$$

(1) Sketch the curve and if possible, find a Cartesian equation for the astroid.
(2) Find the tangent to the astroid in terms of θ (How about in terms in x). How does the tangent change as θ varies? When does the tangent horizontal or vertical?
(3) Find the area of the region enclosed by the astroid.
(4) Find the length of the astroid.
(5) Find the area of the surface generated by rotating the astroid about x-axis.
(6)(Bonus) Change the power from 3 to some other positive integers n and do the same.

Polar Coordinates

The polar coordinates of the plane is given by (r, θ). The relation between the polar coordinate and the Cartesian coordinate is

$$x = r\cos\theta, y = r\sin\theta; \qquad r^2 = x^2 + y^2, \tan\theta = \frac{y}{x}.$$

In the plane, we need two parameters to determine a position.
Usually, we consider the angel as the parameter and use the parametric equation method to deal with the problems.
How to draw the curve with polar equation?
Excise:
1. Change the following polar equations to Cartesian equations and graph the curves.
(1) $r = \csc\theta$; (2) $r = \tan\theta\sec\theta$; (3) $r^2 - 3r + 2 = 0$.

2. Change the following Cartesian equations to polar equations and graph the curves.
(1) $x + y = 1$; (2) $xy = 1$; (3) $y = x^2 - \frac{1}{4}$.

3. (1) The Archimedean spiral is defined by polar equation $r = a\theta + b$. Graph the Archimedean spiral for $a = 1, b = 0$. Find the tangent line in terms of θ. When is the tangent vertical or horizontal? Find the length when $0 \leq \theta \leq 2\pi$. Find the area bounded by the Archimedean spiral and lie in the section $0 \leq \theta \leq 2\pi$.