Math 54: Worksheet

January 31

(1) Solve the sytem

$$\begin{cases} x_1 + 3x_2 - 5x_3 = 4 \\ x_1 + 4x_2 - 8x_3 = 7 \\ -3x_1 - 7x_2 + 9x_3 = -6. \end{cases}$$

Give a geometric interpretation of the solution. Are there other equivalent ways to write down your solution? Can you characterize all possible ways the solution could be written down? (Hint: your answer should have to do with the corresponding *homogeneous* system.)

(2) What is the difference between a matrix of size $m \times n$ and a linear transformation from $\mathbb{R}^n$ to $\mathbb{R}^m$? Given such a matrix, how do we define a corresponding linear transformation? Given such a linear transformation, how do we define a corresponding matrix?

(3) Can there exist a linear transformation $T : \mathbb{R}^2 \to \mathbb{R}^2$ that satisfies $T\left(\begin{bmatrix} 1 \\ 2 \end{bmatrix}\right) = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ and $T\left(\begin{bmatrix} 1 \\ 1 \end{bmatrix}\right) = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$?

(4) Can there exist a linear transformation $T : \mathbb{R}^2 \to \mathbb{R}^2$ that satisfies $T\left(\begin{bmatrix} 1 \\ 2 \end{bmatrix}\right) = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$, $T\left(\begin{bmatrix} 1 \\ 1 \end{bmatrix}\right) = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$, and $T\left(\begin{bmatrix} 0 \\ 1 \end{bmatrix}\right) = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$?

(5) Can there exist a linear transformation $T : \mathbb{R}^2 \to \mathbb{R}^2$ that satisfies $T\left(\begin{bmatrix} 1 \\ 2 \end{bmatrix}\right) = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$, $T\left(\begin{bmatrix} 1 \\ 1 \end{bmatrix}\right) = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$, and $T\left(\begin{bmatrix} 0 \\ 1 \end{bmatrix}\right) = \begin{bmatrix} 0 \\ -1 \end{bmatrix}$?

(6) Suppose that $\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3$ are vectors in $\mathbb{R}^3$ and your friend tells you the values $T(\mathbf{u}_1)$, $T(\mathbf{u}_2)$, and $T(\mathbf{u}_3)$ of a linear transformation $T : \mathbb{R}^3 \to \mathbb{R}^2$. First of all, is it possible that there does not actually exist a linear transformation with these values? If not, has your friend given you enough information to determine the linear transformation uniquely?

(7) Suppose that A is an $m \times n$ matrix and the system $A\mathbf{x} = \mathbf{b}$ is consistent for all $\mathbf{b} \in \mathbb{R}^m$. Which of the following are possible: $m < n$, $m = n$, $m > n$.

(8) Suppose that A is an $m \times n$ matrix and the system $A\mathbf{x} = \mathbf{b}$ is has a unique solution for all $\mathbf{b} \in \mathbb{R}^m$. Which of the following are possible: $m < n$, $m = n$, $m > n$.

(9) **True or false:** if A is an $m \times n$ matrix with $n > m$ and $\mathbf{b} \in \mathbb{R}^m$, then the system $A\mathbf{x} = \mathbf{b}$ has infinitely many solutions.

(10) **True or false:** if A is an $m \times n$ matrix with $n > m$, then the system $A\mathbf{x} = \mathbf{0}$ has infinitely many solutions.

(11) Suppose that A is an $m \times n$ matrix with $n > m$. Can the corresponding linear transformation T_A be onto but not one-to-one? Can it be one-to-one but not onto? Can it be both? Can it be neither?

(12) Same question but for $n < m$.

(13) Same question but for $n = m$.