# Math 54: Quiz #1 Solutions

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Please give neat and organized answers. Whenever applicable (especially for computational questions), make it clear what strategy you are using.

## Problem 1

For what values of $h$ is $\begin{bmatrix} 1 & h & -3 \\ -2 & 4 & 8 \end{bmatrix}$ the augmented matrix of a consistent linear system?

**Solution:** First put the matrix into echelon form:

$$\begin{bmatrix} 1 & h & -3 \\ -2 & 4 & 8 \end{bmatrix} \xrightarrow{R_2 \leftarrow \frac{1}{2} R_2} \begin{bmatrix} 1 & h & -3 \\ -1 & 2 & 4 \end{bmatrix}$$
$$\xrightarrow{R_2 \leftarrow R_2 + R_1} \begin{bmatrix} 1 & h & -3 \\ 0 & h+2 & 1 \end{bmatrix}.$$

If $h = -2$, then the last column is a pivot column, so the linear system corresponding to the matrix is inconsistent. Otherwise, only the first two columns are pivot columns, so the corresponding linear system is consistent. Thus the answer is: all values except $h = -2$.

## Problem 2

Solve the following linear system:

$$\begin{cases} 4x_1 + 3x_2 + 2x_3 + x_4 = 0 \\ 9x_1 + 8x_2 + 7x_3 + 6x_4 = 5 \\ 5x_1 + 3x_2 + 2x_3 + x_4 = 0 \end{cases}$$

**Solution:**

$$\begin{bmatrix} 4 & 3 & 2 & 1 & 0 \\ 9 & 8 & 7 & 6 & 5 \\ 5 & 3 & 2 & 1 & 0 \end{bmatrix} \xrightarrow{R_1 \leftrightarrow R_3} \begin{bmatrix} 5 & 3 & 2 & 1 & 0 \\ 9 & 8 & 7 & 6 & 5 \\ 4 & 3 & 2 & 1 & 0 \end{bmatrix}$$

$$\xrightarrow{R_1 \leftarrow R_1 - R_3} \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 9 & 8 & 7 & 6 & 5 \\ 4 & 3 & 2 & 1 & 0 \end{bmatrix}$$

$$\xrightarrow{R_2 \leftarrow R_2 - 9R_1} \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 8 & 7 & 6 & 5 \\ 4 & 3 & 2 & 1 & 0 \end{bmatrix}$$

$$\xrightarrow{R_3 \leftarrow R_3 - 4R_1} \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 8 & 7 & 6 & 5 \\ 0 & 3 & 2 & 1 & 0 \end{bmatrix}$$

$$\xrightarrow{R_2 \leftarrow R_2 - R_3} \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 5 & 5 & 5 & 5 \\ 0 & 3 & 2 & 1 & 0 \end{bmatrix}$$

$$\xrightarrow{R_2 \leftarrow \frac{1}{5} R_2} \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 1 \\ 0 & 3 & 2 & 1 & 0 \end{bmatrix}$$

$$\xrightarrow{R_3 \leftarrow R_3 - 3R_2} \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & -1 & -2 & -3 \end{bmatrix} \quad \text{(echelon form)}$$

$$\xrightarrow{R_3 \leftarrow -R_3} \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 2 & 3 \end{bmatrix}$$

$$\xrightarrow{R_2 \leftarrow R_2 - R_3} \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 & -2 \\ 0 & 0 & 1 & 2 & 3 \end{bmatrix} \quad \text{(reduced echelon form)}$$

The first three columns are pivot columns, and the fourth column is a free column. Thus the solution is given by

$$\begin{cases} x_1 = 0 \\ x_2 = x_4 - 2 \\ x_3 = -2x_4 + 3 \\ x_4 \text{ free} \end{cases}$$