Math 54: Fourier cosine and sine series

May 1

Suppose that f is a (piecewise continuous) function on $[0, L]$. This is different from the setting of the ordinary Fourier series, in which we considered functions on $[-L, L]$. The **Fourier cosine series** represents f as a sum of the even Fourier modes, i.e.,

$$\boxed{f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\left(\frac{n\pi x}{L}\right),}$$

where

$$\boxed{a_n = \frac{2}{L}\int_0^L f(x)\cos\left(\frac{n\pi x}{L}\right)\,dx \text{ for } n = 0, 1, \ldots}$$

(The Fourier cosine series can be derived by extending f to an **even** function on $[-L, L]$, then computing the ordinary Fourier series for this function on this symmetric interval. Since f is even on the enlarged interval, all of the b_n coefficients will be zero, and the a_n coefficients will be given by the above formula. See my handout on Fourier series for more details.)

Now if f is again a (piecewise continuous) function on $[0, L]$, the **Fourier sine series** represents f as a sum of the odd Fourier modes, i.e.,

$$\boxed{f(x) = \sum_{n=1}^{\infty} b_n \sin\left(\frac{n\pi x}{L}\right),}$$

where

$$\boxed{b_n = \frac{2}{L}\int_0^L f(x)\sin\left(\frac{n\pi x}{L}\right)\,dx \text{ for } n = 1, 2, \ldots}$$

(The Fourier sine series can be derived by extending f to an **odd** function on $[-L, L]$, then computing the ordinary Fourier series for this function on this symmetric interval. Since f is odd on the enlarged interval, all of the a_n coefficients will be zero, and the b_n coefficients will be given by the above formula.)

To compute the Fourier cosine and sine series for some given function f, you need only plug f into the above formulas and compute the appropriate integrals. Also see page 589 in the textbook.