Math 53: Quiz #3

February 22

GSI: M. Lindsey

20 points, 20 minutes

Name: ______________________________________

Please give neat and organized answers. Whenever applicable (especially for computational questions), make it clear what strategy you are using. Points may be deducted for poor exposition.

Problem 1

(10 points.) Consider a function $f(x, y)$ of two variables, and fix a point (x_0, y_0).

(a) Write down a formula for the function f_{lin} that linearly approximates f about the point (x_0, y_0). (This formula should involve the partial derivatives of f.)

(b) The graph of $z = f_{\text{lin}}(x, y)$ is a plane in three-dimensional space. Using your formula from part (a), find a normal vector to this plane.

(c) Recall that the plane from part (b) is the tangent plane to the graph of f at the point $(x_0, y_0, f(x_0, y_0))$. Suppose that the tangent planes to the graph of f at ALL points (x_0, y_0) are parallel and that $f(0, 0) = 5$, $\frac{\partial f}{\partial x}(0, 1) = \frac{\partial f}{\partial y}(1, 0) = 0$. Based on part (b), write down an explicit formula for $f(x, y)$.

(See back for next problem!)

Problem 2

(10 points.) If $z = f(x, y)$ where $x = r + s^3$ and $y = e^{rs}$, find $\frac{\partial^2 z}{\partial r \partial s}$. (Your answer should be expressed in terms of r, s, and partial derivatives of z with respect to x and y.)