

1. Find all solutions for below equations.

a.

$$y''(t) + 7y'(t) + 10y(t) = 0$$

$$\text{Answer. } c_1 e^{-2t} + c_2 e^{-5t}$$

b.

$$y''(t) + 2y'(t) - 3y(t) = 0$$

$$\text{Answer. } c_1 e^t + c_2 e^{-3t}$$

c.

$$y''(t) - 4y(t) = 0, \quad y(0) = 4, \quad y'(0) = -4$$

$$\text{Answer. } e^{2t} + 3e^{-2t}$$

d.

$$y''(t) - 8y'(t) + 16y(t) = 0, \quad y(1) = 3, \quad y'(1) = 13$$

$$\text{Answer. } 3e^{4(t-1)} + (t-1)e^{4(t-1)}$$

e.

$$y''(t) - 4y'(t) + 5y(t) = 0$$

$$\text{Answer. } c_1 e^{2t} \cos t + c_2 e^{2t} \sin t$$

f.

$$y''(t) - 6y'(t) + 10y(t) = 0, \quad y(-1) = 2, \quad y'(-1) = 9$$

$$\text{Answer. } 2e^{3(t+1)} \cos(t+1) + 3e^{3(t+1)} \sin(t+1)$$

g.

$$y''(t) - 3y'(t) + 2y(t) = 2 \sin 2t$$

$$\text{Answer. } 3 \cos 2t - \sin 2t + c_1 e^t + c_2 e^{2t}$$

h.

$$y''(t) - 2y'(t) + 5y(t) = -5t^3 - t^2 - 10t + 2$$

$$\text{Answer. } -t^3 + t^2 + c_1 e^t \cos 2t + c_2 e^t \sin 2t$$

i.

$$y''(t) - 2y'(t) + y(t) = -8 \cos 3t + 6 \sin 3t$$

$$\text{Answer. } \cos 3t + c_1 e^t + c_2 t e^t$$

j.

$$y''(t) - y'(t) - 2y(t) = 2t^4 + 4t^3 - 12t^2$$

$$\text{Answer. } -t^4 + c_1 e^{-t} + c_2 e^{2t}$$

k.

$$y''(t) - 2y'(t) + 2y(t) = t^2(\cos t + 2 \sin t) - 4t(\cos t + \sin t) + 2 \cos t$$

$$\text{Answer. } t^2 \cos t + c_1 e^t \cos t + c_2 e^t \sin t$$

2. Find all solutions for below equations.

a.

$$t^2 y''(t) - 5ty'(t) + 8y(t) = 0$$

$$\text{Answer. } c_1 t^4 + c_2 t^2$$

b.

$$t^2 y''(t) + 7ty'(t) + 8y(t) = 0$$

$$\text{Answer. } c_1 t^{-4} + c_2 t^{-2}$$

c.

$$t^2 y''(t) - 3ty'(t) + 4y(t) = 0$$

$$\text{Answer. } c_1 t^2 + c_2 t^2 \ln t$$

d.

$$t^2 y''(t) - 2y(t) = 4t^3$$

$$\text{Answer. } t^3 + c_1 t + c_2 t^{-2}$$

e.

$$t^2 y''(t) + ty'(t) + y(t) = 8t^3 - 1$$

$$\text{Answer. } t^3 - t + 1 + c_1 t + c_2 t^{-1}$$

f.

$$t^2 y''(t) + 2ty'(t) - 2y(t) = 10t^3$$

$$\text{Answer. } t^3 + c_1 t^2 + c_2 t^{-1}$$

g.

$$t^2 y''(t) - 5ty'(t) + 9y(t) = 0, \quad y(1) = 4, \quad y'(1) = 9$$

$$\text{Answer. } 4t^3 - 3t^3 \ln t$$

h.

$$t^2 y''(t) + 3ty'(t) + 5y(t) = 0$$

$$\text{Answer. } c_1 t \cos(2 \ln t) + c_2 t \sin(2 \ln t)$$

i.

$$t^2 y''(t) - ty'(t) + 10y(t) = 10t^2 + 9t + 10$$

$$\text{Answer. } t^2 + t + 1 + c_1 t^{-1} \cos(3 \ln t) + c_2 t^{-1} \sin(3 \ln t)$$

3. Find one solution  $y(t)$  satisfying each equation.

a.

$$y''(t) - 6y(t)^{1/3} = 0$$

Answer.  $t^3$

b.

$$y''(t) - y(t) = 0$$

Answer.  $e^t$

c.

$$y''(t) - 2y(t)^3 - 2y(t) = 0$$

Answer.  $\tan t$

d.

$$y''(t) + y(t) = 0, \quad y(0) = 1, \quad y'(0) = 0$$

Answer.  $\cos t$

e.

$$y''(t) + y(t) = 0, \quad y(0) = 0, \quad y'(0) = 1$$

Answer.  $\sin t$

f.

$$y''(t) - 6y(t)^2 = 0$$

Answer.  $\frac{1}{(1-t)^2}$

g.

$$y''(t) + e^{-2y(t)} = 0$$

Answer.  $\ln t$

h.

$$y''(t) + \frac{1}{4y(t)^3} = 0$$

Answer.  $\sqrt{t}$

Note that

$$\int \frac{1}{1+y^2} = \arctan y$$

4. Find all solutions for below equations.

a.

$$y''''(t) - 7y'''(t) + 19y''(t) - 23y'(t) + 10y(t) = 0$$

$$\text{Answer. } c_1 e^t + c_2 e^{2t} + c_3 e^{2t} \cos t + c_4 e^{2t} \sin t$$

b.

$$y''''(t) - 8y'''(t) + 23y''(t) - 30y'(t) + 18y(t) = 0$$

$$\text{Answer. } c_1 e^{3t} + c_2 t e^{3t} + c_3 e^t \cos t + c_4 e^t \sin t$$

c.

$$y''''(t) - 6y'''(t) + 11y''(t) - 6y'(t) = 0$$

$$\text{Answer. } c_1 + c_2 e^t + c_3 e^{2t} + c_4 e^{3t}$$

d.

$$y''''(t) - 6y''(t) - 8y'(t) - 3y(t) = 0$$

$$\text{Answer. } c_1 e^{-t} + c_2 t e^{-t} + c_3 t^2 e^{-t} + c_4 e^{2t}$$

e.

$$t^3 y'''(t) + t y'(t) - y(t) = 0$$

$$\text{Answer. } c_1 t + c_2 t \ln t + c_3 t (\ln t)^2$$

f.

$$t^3 y'''(t) - 3t^2 y''(t) + 6t y'(t) - 6y(t) = 0$$

$$\text{Answer. } c_1 t + c_2 t^2 + c_3 t^3$$