1. Given that $y_1(t) = \frac{1}{4}\sin 2t$ is a solution to

$$y'' + 2y' + 4y = \cos 2t$$

and that $y_2(t) = \frac{t}{4} - \frac{1}{8}$ is a solution to

$$y'' + 2y' + 4y = t.$$

Find solutions to the following:

(a) $y'' + 2y' + 4y = t + \cos 2t$

(b) $y'' + 2y' + 4y = 2t - 3\cos 2t$

(c) $y'' + 2y' + 4y = 11t - 12\cos 2t$

2. Find a general solution to the differential equation using the method of variation of parameters.

(a) $y'' + 4y = \tan 2t$

(b) $y'' + 4y' + 4y = e^{-2t}\ln t$

3. Use the energy integral lemma to show that motions of the free undamped mass-spring oscilator $my'' + ky = 0$ obey

$$m(y')^2 + ky^2 = \text{constant}$$

4. Show that the three solutions

$$\frac{1}{(1-t)^2}, \quad \frac{1}{(2-t)^2}, \quad \frac{1}{(3-t)^2}$$

to

$$y'' - 6y^2 = 0$$

are linearly independent on $(-1, 1)$.