1 The concept of (Linear or matrix) Transformation

1.1 Matrix transformation

A transformation T from $\mathbb{R}^n$ to $\mathbb{R}^m$ is a mapping that assigns each vector $\mathbf{x}$ in $\mathbb{R}^n$ a vector $T(\mathbf{x})$ in $\mathbb{R}^m$. When $T(\mathbf{x})$ turns out to be $A\mathbf{x}$, where A is an $m \times n$ matrix, we denotes the transformation $\mathbf{x} \mapsto A\mathbf{x}$ as a *matrix transformation.*

1.2 Linear transformation

There are important two properties that every matrix transformation has in common.

$$A(\mathbf{x}+\mathbf{y}) = A\mathbf{x} + A\mathbf{y}, \qquad A(c\mathbf{x}) = cA\mathbf{x}\ (\forall c \in \mathbb{R})$$

So, from now on, we want to think about all kinds of transformations that satisfy above two properties and such transformations are called ***Linear Transformations***.

1.2.1 Definition of a Linear Transformation

A transformation $T : \mathbb{R}^n \to \mathbb{R}^m$ is called a *linear transformation* when T satisfies

$$T(\mathbf{x}+\mathbf{y}) = T(\mathbf{x}) + T(\mathbf{y}), \qquad T(c\mathbf{x}) = cT(\mathbf{x})\ (\forall c \in \mathbb{R})$$

1.2.2 Linear transformation vs. Matrix transformation

It turns out to be that every linear transformation $T : \mathbb{R}^n \to \mathbb{R}^m$ is exactly a transformation that multiplies a $m \times n$ matrix A when it is applied a vector in $\mathbb{R}^n$. Here is Theorem 10 in Lay.

Theorem 10. Let $T : \mathbb{R}^n \to \mathbb{R}^m$ be a linear transformation. Then there exists a unique matrix A such that

$$T(\mathbf{x}) = A\mathbf{x} \text{ for all } \mathbf{x} \text{ in } \mathbb{R}^n$$

We call the matrix A as *the standard matrix for T*.

1.2.3 Onto mappings and one-to-one mappings

When is a mapping $T : \mathbb{R}^n \to \mathbb{R}^m$ called ***onto***? and ***one-to-one***?

2 Matrix Algebra

2.1 Matrix operations

Sum $A + B$ Scalar multiple cA Matrix multiplication AB

There is one important condition that two matrices A and B should satisfy to make sense of AB. What is it?

2.2 Properties of Matrix multiplication

2.3 Powers of a Matrix and the Transpose of a Matrix

2.4 the Inverse matrix

What is the *inverse matrix* of a matrix A? When is it defined well?

Why is the *inverse matrix* important?

We use notation A^{-1} to represent the *inverse matrix* of a matrix A.