1 Solutions of a linear system

1.1 Concepts

What is a ***linear equation***? and a ***linear system***?

A *linear system* is said to be **consistent** if it has either one solution or infinitely many solutions; a system is **inconsistent** otherwise (= if it has no solution).

1.2 One important tool ; Matrix

Given a *linear system*, we define the ***coefficient matrix*** and the ***augmented matrix***.

1.3 Row operations

One way to solve a linear system is 'finding a solution using ***elementary row operations***'.

1) *Replacement* ; Add to one row a multiple of another row

2) *Interchange* ; Interchange two rows

3) *Scaling* ; Multiply all entries in a row by a nonzero constant

2 Row reduction and Echelon Forms

$*$ Two matrices are called **row equivalent** to each other if one of them is induced from the other by elementary row operations.

2.1 Echelon Form

A rectangular matrix is in **echelon form** if it has the following three properties:

(1) All nonzero rows are above any rows of all zeros.

(2) Each leading entry of a row is in a column to the right of the leading entry of the row above it.

(3) All entries in a column below a leading entry are zeros.

For a given matrix, there are two concepts to remember ; an ***echelon form*** and the ***reduced echelon form***.

> An *echelon form of a given matrix* is a matrix in *echelon form* which is *row equivalent* to the given matrix. The *reduced echelon form of a given matrix* is the matrix in *reduced echelon form* which is *row equivalent* to the given matrix. Note that there exists a unique *reduced echelon form* for a given matrix.

2.2 Pivot positions

What is a ***pivot position***?

3 Vectors

3.1 What is a vector?

We have two operations for vectors ; **sum** and **scalar multiplication**

3.2 Linear combinations

What is a ***linear combination*** of vectors $\{v_1, v_2, \cdots, v_n\}$?

3.3 Span

What is ***Span***$\{v_1, \cdots, v_n\}$?