1. Use the Chain Rule to find $dz/dt$ for a) and $\partial z/\partial s$, $\partial z/\partial t$ for b).

a) $z = \frac{x^3 - x \ln y + y}{\sin y}, \quad x = e^t, \quad y = t^2$

Answer. $\frac{e^t(3e^{2t} - 2 \ln t)}{\sin(t^2)} + 2t \frac{\sin(t^2)(1 - \frac{e^t}{t^2}) - \cos(t^2)(e^{3t} - 2e^t \ln t + t^2)}{\sin^2(t^2)}$

b) $z = x \sin \theta, \quad x = \frac{s}{t}, \quad \theta = s^2 + t$

Answer. $\frac{\partial z}{\partial s} = \frac{1}{t} \sin(s^2 + t) + \frac{2s^2}{t} \cos(s^2 + t)$

$\frac{\partial z}{\partial t} = -\frac{s}{t^2} \sin(s^2 + t) + \frac{s}{t} \cos(s^2 + t)$

2. Find $dy/dx$ for a) and $\partial z/\partial x$, $\partial z/\partial y$ for b) and c).

a) $x^2 + \sin x \sin y - y^2 = 0$

**Solution.** Using the formula for implicit differentiation

$$\frac{dy}{dx} = -\frac{F_x}{F_y},$$

we get the answer.

**Answer.** $\frac{dy}{dx} = 2x + \cos x \sin y \quad 2y - \sin x \cos y$

b) $x + y^2 + z^3 = 0$

**Solution.** Using the formula for implicit differentiation

$$\frac{\partial z}{\partial x} = -\frac{F_x}{F_z} \quad \& \quad \frac{\partial z}{\partial y} = -\frac{F_y}{F_z},$$

we get the answer.

**Answer.** $\frac{\partial z}{\partial x} = -\frac{1}{3z^2}, \quad \frac{\partial z}{\partial y} = \frac{2y}{3z^2}$
c) \( \tan x + e^y + z^3 - z^2 = 0 \)

**Solution.** Same as before.

**Answer.** \( \frac{\partial z}{\partial x} = -\frac{1}{z(3z - 2) \cos^2 x}, \quad \frac{\partial z}{\partial y} = -\frac{e^y}{z(3z - 2)} \)

3. Find the gradient of \( f \), evaluate the gradient at the point \( P \), and find the rate of change of \( f \) at \( P \) in the direction of the vector \( u \).

\[
f(x, y, z) = \cos(x^2) + xy + \ln z, \quad P = (\pi, 1, e) \quad u = \left\langle \frac{1}{9}, -\frac{8}{9}, \frac{4}{9} \right\rangle
\]

**Solution.** The gradient of \( f \) is defined as following :

\[
\nabla f = \left( \frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z} \right).
\]

Hence, in this problem, \( \nabla f = (-2x \sin(x^2) + y, x, \frac{1}{z}) \). At the point \( P \),

\[
\nabla f(P) = (-2\pi \sin(\pi^2) + 1, \pi, \frac{1}{e}).
\]

The rate of change of \( f \) at \( P \) in the direction of the vector \( u \) is given as

\[
\nabla f(P) \cdot u = \frac{1}{9} \left( -2\pi \sin(\pi^2) + 1 - 8\pi + \frac{4}{e} \right).
\]

**Answer.** Answers are listed in the **Solution.**

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Letter grade for Quiz 6

\[
\begin{array}{ll}
A^+ = 20 & (6) \\
A0 = 19 & (6) \\
B^+ = 18 & (9) \\
B0 = 16, 17 & (6) \\
C^+ = .. & (2)
\end{array}
\]