**Quiz 11** (20MINS, 30PTS)

Please write down your name, SID, and solutions discernably.

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SID:  
Score: 

1. (10pts) Evaluate the line integral.

\[ \int_C xye^{xy} \, dy, \]

where \( C : x = t, \ y = t^2, \ z = t^3, \ 0 \leq t \leq 1. \)

We are given a parametrisation of \( C. \)

By definition, \( \int_C y \, dy, \)

\[ \int_C xye^{xy} \, dy = \int_0^1 t \cdot t^2 \cdot e^{t^3} \cdot 2t \, dt = \int_0^1 2t^4 \cdot e^{t^3} \, dt \]

\[ = \frac{1}{3} \left( e^{t^3} \right)_0^1 = \frac{1}{3} (e - 1) \]

Answer: \( \frac{1}{3} (e - 1) \)

2. (10pts) Evaluate the line integral \( \int_C \mathbf{F} \cdot d\mathbf{r}. \)

\( \mathbf{F}(x, y) = (x - y)\mathbf{i} + y^2\mathbf{j} + (z - x)\mathbf{k}, \)

where \( C \) is given by the vector function \( \mathbf{r}(t) = t^2\mathbf{i} - t^2\mathbf{j} + tk, \ 0 \leq t \leq 1. \)

\[ \int_C \mathbf{F} \cdot d\mathbf{r} = \int_0^1 (t^3 - t^2 + t^3, t - t, t^2 - 2t) \cdot (3t^2 - 2t, 1, k) \, dt \]

\[ = \int_0^1 (3t^5 - 3t^4 + 3t^3 - 2t^2 + t - t) \, dt = \int_0^1 3t^5 + 3t^3 + 3t^2 - 4t \, dt \]

\[ = \left[ \frac{3}{6} t^6 + \frac{3}{4} t^4 + \frac{3}{3} t^3 - \frac{4}{2} t^2 \right]_0^1 \]

\[ = \frac{3}{6} + \frac{3}{4} - \frac{4}{2} = 0 \]

Answer: \( 0 \)
3. (10pts) Find a function $f$ such that $\mathbf{F} = \nabla f$ and use $f$ to evaluate $\int_C \mathbf{F} \cdot d\mathbf{r}$ along the given curve $C$.

$$\mathbf{F}(x, y) = (1 + xy)e^{xy}i + x^2e^{xy}j, \quad C: \mathbf{r}(t) = \cos t + 2 \sin t j, \quad 0 \leq t \leq \frac{\pi}{2}$$

From the fact that $\frac{\partial f}{\partial y} = xe^{xy}$, we might guess $f(x, y) = xe^{xy}$ as a function of $x$.

But, it turns out that $f(x, y) = xe^{xy}$ satisfies $\mathbf{F}(x, y) = \nabla f(x, y)$.

Obviously, $f$ is defined on the whole plane $\mathbb{R}^2$ which is simply connected. Hence, we can apply Fundamental Theorem of Line Integrals, so that

$$\int_C \mathbf{F} \cdot d\mathbf{r} = f(\mathbf{r}(\pi)) - f(0) = f(0, 2) - f(1, 0) = 0 - 1 \cdot e^0 = -1.$$

Answer. $-1$. 