MATH 1B: CALCULUS DISCUSSION SECTION 2: HAPPY HALLOWEEN!

Tricks for Dealing With Series:

1. Read the problem CAREFULLY. Make sure you notice where the series starts, where the parenthesis are, if there are quantities which are not yet defined (e.g. do I need to determine the values of p for which this converges?).
2. Think about what this looks like that you've seen before. Maybe you can make it look like a problem you have already done!
3. Make an educated guess as to whether its convergent or divergent. Use this to decide what test to use, and how to use it.
4. Sometimes it helps to re-write expressions with powers in them (e.g. $n^{\ln n} = e^{(\ln n)^2}$) so you can see more clearly what's going on.
5. Split up sum into two pieces IF THEY ARE BOTH CONVERGENT!. This is nice if the terms of the series look different for, say, odd and even n.
6. Write out the first two or three terms or, conversely, if you only know about the first couple of terms, see if you can generalize to figure out what the n^{th} term is.
7. Gather your evidence and CONSTRUCT AN ARGUMENT. Write down which test you are using, why, and what the result is. If the result is inconclusive, ask yourself why, and use the answer to motivate a better test.

Your tricks here:

Some Scary Series:

First, determine what test you would use to prove the following series are convergent or divergent. Then, go back and prove your hypothesis. (* indicates that you should find the sum of the series).

(a) $^* \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n2^{n+1}}$

(b) $\sum_{n=1}^{\infty}(-1)^n \frac{x^n}{3^{2n}}$

(c) $\sum_{n=1}^{\infty}(-1)^n \tan^{-1}\left(\frac{1}{n}\right)$

(d) $^* \sum_{n=0}^{\infty} \left(\frac{1}{1+3\cdot(-1)^n}\right)^n$

(e) $^* \sum_{n=1}^{\infty} \frac{(-1)^n \pi^{2n}}{n!}$

(f) $\sum_{n=1}^{\infty} \frac{\sin(1/n)}{n^{0.3}}$

(g) $\sum_{n=1}^{\infty} ne^{-n^2}$

(h) $\sum_{n=2}^{\infty} \frac{n}{(\ln(n))^{\ln(n)}}$

(i) $\sum_{n=1}^{\infty} \left(\frac{n}{n+1}\right)^{n^3}$

(j) $\sum_{n=1}^{\infty} \frac{1}{n^{1+1/n}}$

(k) $\sum_{n=1}^{\infty}(-1)^n \left(\sin(1/n^2)\right)^{1/3}$

(l) $\sum_{n=1}^{\infty} \frac{\sqrt{n+1}-\sqrt{n-1}}{n}$

(m) $\sum_{n=1}^{\infty} \frac{(-1)^n\sqrt{n}}{n+5}$

Classifying Tests for Convergence and Divergence

Test name	*Example of series to test*	*Conditions on series to be tested*	*Conclusions you can draw from this test*
p-Series			
Geometric Series			
Comparison			
Limit Comparison			
Alternating Series			
Divergence			
Integral			
Root			
Ratio			

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