MATH 1B: CALCULUS DISCUSSION SECTION 2

WORKSHEET 8

Fun with Taylor and Maclaurin Series

1. In special relativity, the momentum p of an object of mass m moving at speed v is given by

$$p = \frac{mv}{\sqrt{1 - \left(\frac{v}{c}\right)^2}}$$

where c is the speed of light. Find the first two terms of a Taylor expansion. If $v << c$, what is the expression for momentum? What is the radius of convergence? If we assert that this law is true for any moving object, what does this tell you about the maximum speed at which something can travel?

2. Find the first two terms of a Taylor Series representation for the following. Estimate the size of the error. What is the radius of convergence? Can you find an expression for the n^{th} term?

   (a)
   $$f(x) = x^{1/3} \quad \text{about } x = 2$$

   (b)
   $$g(x) = x^2 + 2x + 1 \quad \text{about } x = 1$$

   (c)
   $$h(x) = \cos^2(x) \quad \text{about } x = 0$$

3. Find the value of

   (a)
   $$\sum_{n=0} \frac{(-1)^n \pi^n}{n!}$$

   (b)
   $$\sum_{n=0} \frac{(-1)^n \pi^{2n}}{9^n (2n)!}$$

4. Find a Maclaurin series for the function

$$f(x) = \begin{cases} e^{-\frac{1}{x^2}} & x \neq 0 \\ 0 & x = 0 \end{cases}$$

   What is the radius of convergence?

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