MATH 1B: CALCULUS DISCUSSION SECTION 2

WORKSHEET 5

1. Suppose that you are a very contrary person and you like to come up with the logical negative of any statement someone else makes. For each of the following statements, write down the logical negative and then figure out what evidence you would need to prove the logical negative of the statement.

   (a) **Example** If it is an accordian, then it is orange.
   Negative: *There exist accordians which are not orange.* ***Proof***: *Find an accordian which is not orange*.

   (b) If it is a snake, then it might have polka dots.

   (c) If it is a mathematician, then it is wearing purple shoes.

   (d) If $\lim_{n\to\infty} a_n = L$, then for any positive number ϵ, we can find some positive integer N such that $|a_n - L| < \epsilon$ whenever $n > N$.

   (e) **Bonus!**Bob Marley was killed by the C.I.A., which directed an operative to give him a pair of boots which contained a copper wire inside that would poke his foot as he wore them, eventually causing cancer and death.[1]

2. Decide if the following limit rules are true or false. If false, find a counterexample.

   (i) If $\lim_{n\to\infty}(a_n + b_n) = L$, then there are numbers L_1 and L_2 such that $\lim_{n\to\infty} a_n = L_1$ and $\lim_{n\to\infty} b_n = L_2$, where $L_1 + L_2 = L$.

   (ii) If $\lim_{x\to\infty} f(x) = L$, then $\lim_{n\to\infty} f(n) = L$.

   (iii) If $\lim_{n\to\infty} a_n = L$, then $\lim_{n\to\infty} f(a_n) = f(L)$.

   (iv) $\lim_{n\to\infty} \frac{a_n}{b_n} = \frac{\lim_{n\to\infty} a_n}{\lim_{n\to\infty} b_n}$

[1] I did not make this up...it comes from the wikipedia list of popular conspiracy theories!

3. Given an example of a sequence which is...
    (I) ...bounded but not convergent
    (II) ...monotone but not bounded
    (III) ...monotone and bounded but not convergent

4. **Zeno's Paradox** Your friend dares you to walk across the room in the following way: take a step which covers half the distance, then a step which covers one-fourth of the distance, then a step covering one-eighth... Another friend, Zeno, tells you that this is a preposterous way to walk across the room because there is no way for you to take an infinite number of steps.
    (a) Write down a sequence a_n where each term is the distance you cover on the n^{th} step.
    (b) Write down a sequence b_n where each term is the total distance you have covered after n steps.
    (c) Is Zeno right or wrong? Can you cross the room this way?

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