MATH 1B: CALCULUS DISCUSSION SECTION 2

WORKSHEET 4

1. **(a)** Evaluate the integral

$$\int_{-\infty}^{\infty} x \, dx$$

by considering

$$\lim_{t\to\infty} \int_{-t}^{t} x \, dx$$

(b) Is this integral convergent? What is going on here?

2. For each of the integrals below, determine what type of improper integral it is, and whether or not it converges.

(a) $\displaystyle\int_{\pi/4}^{3\pi/4} \tan(x)dx$

(b) $\displaystyle\int_{0}^{\infty} \frac{\sin(x)}{x}dx$

(c) $\displaystyle\int_{0}^{10^{100}} \ln(x)dx$

3. The amount of electromagnetic radiation emitted by a blackbody (like your oven!) is found by integrating the *spectrum*[1] over all frequencies of photons. The Rayleigh-Jeans spectrum is

$$s(x) = K_r x^2$$

The Planck spectrum is

$$s(x) = K_p \frac{x^3}{e^x - 1}$$

If photon frequencies range from zero to infinity, what does each theory predict for the energy $E = \int_0^\infty s(x)dx$ Which theory do you trust? [2]

4. For what values of p is the following convergent?

$$\int_0^1 \frac{dx}{x^p}$$

5. **(a)** Find

$$\lim_{x\to\infty} x\sin(x^3)$$

(b) Is

$$\int_1^\infty x\sin(x^3)dx$$

a convergent integral? Hint: try integrating by parts with $u = 1/x$ and $dv = x^2\sin(x^3)$.

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[1] which essentially tells you the energy radiated per unit photon photon energy

[2] The divergence of the classical Rayleigh-Jeans theory is called the "ultraviolet catastrophe" and helped lead to the development of quantum mechanics!