MATH 1B: CALCULUS DISCUSSION SECTION 2

WORKSHEET 2

SUPER DUPER USEFUL TRIGONOMETRIC IDENTITIES YOU SHOULD MEMORIZE

$$1 = \sin^2(x) + \cos^2(x) \tag{1}$$

$$\sin(a+b) = \sin(a)\cos(b) + \cos(a)\sin(b) \tag{2}$$

$$\cos(a+b) = \cos(a)\cos(b) - \sin(a)\sin(b) \tag{3}$$

1. Using the equations you have just committed to memory, deduce the following identities:

$$\begin{aligned}
\sin(2x) &= 2\sin(x)\cos(x) \\
\sec^2(x) &= 1 + \tan^2(x) \\
\sin^2(x) &= \frac{1}{2}\left(1 - \cos(2x)\right) \\
\cos^2(x) &= \frac{1}{2}\left(1 + \cos(2x)\right) \\
\sin(a)\sin(b) &= \frac{1}{2}\left(\cos(a-b) - \cos(a+b)\right) \\
\sin(a)\cos(b) &= \frac{1}{2}\left(\sin(a-b) + \sin(a+b)\right) \\
\cos(a)\cos(b) &= \frac{1}{2}\left(\cos(a+b) + \cos(a-b)\right)
\end{aligned}$$

2. The time it takes for a beam of light to travel around a star in curved Schwarzchild spacetime is given by:

$$t = \int \frac{r}{\sqrt{r^2-1}}\left(1+\frac{2M}{r}-\frac{M}{r^3}\right)dr$$

where M is the mass of the star and $r > 1$. [1] Write this integral as the sum of three different integrals and solve them. Hint: if you are flummoxed by terms of the form $\sqrt{r^2-1}$, first do the substitution $u = 1/r$ to get it in a form amenable to trig substitutions.

3. Suppose two particles move with velocities given respectively by the functions

$$v_1(t) = \sin(4t)\cos(t)$$
$$v_2(t) = \sin(6t)\sin(3t)$$

Assume that the particles both start at the origin and find the positions $x_1(t)$ and $x_2(t)$ of each particle as a function of time. Can you draw the path of each particle? At $t = 6.3$, which is farthest away from the origin? (No, you don't need a calculator... think about it in terms of multiples of π...)

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[1] This effect is called the *Shapiro time delay* and is a result of the fact that light "falls" in a gravitational field.