
MATH 1B: CALCULUS DISCUSSION SECTION 2

WORKSHEET 1

1. Give the definition of a *limit* and evaluate

$$(a) \lim_{x \rightarrow 0} \sin(x)/x$$

$$(b) \lim_{x \rightarrow \infty} \cos(x)/x^2$$

2. Write the fundamental theorem of Calculus.

3. Consider the following integration by parts for $\int \frac{1}{x} dx$. Using $u = 1/x$, $du = -1/x^2$, $v = x$, and $dv = 1$, we find:

$$\int \frac{1}{x} dx = 1 + \int \frac{1}{x} dx$$

From which we conclude that $1 = 0$! What went wrong? (Hint: re-write the problem as a definite integral with limits a and b , and think what's wrong with the indefinite integral).¹

4. Use integration by parts to obtain the first two terms of a series (in powers of x) for the *error function*

$$\operatorname{erf}(x) = \left(\frac{2}{\sqrt{\pi}} \right) \int_0^x e^{-t^2} dt$$

Hint: let $u = e^{-t^2}$. **Bonus!** Can you find the third term? The n^{th} ?

5. In lecture you learned an acronym to rank certain types of functions (exponential, logarithmic, inverse trigonometric, trigonometric, and algebraic) in order of which to choose for "u" in the integration by parts formula. *Why?* Write down this ranking. List an example of each type of function and take its derivative.

Bonus! Come up with your own mnemonic to remember the order.

¹Thanks to Peter Mannisto for this cute problem.

²In *Multivariable Calculus* you will learn a trick that allows you to find that $\lim_{x \rightarrow \infty} \operatorname{erf}(x) = 1$.