MATH 128B: WORKSHEET 2

Linear Algebra Review

1. (Here be ~~dragons~~ eigenvalues) Use Gershgorin's circle theorem to draw the region in the complex plane where you will encounter the eigenvalues of the matrix

$$\mathbf{A} = \begin{pmatrix} -1 & 2 & 2 \\ 0 & 3 & 1 \\ 1 & 4 & 1 \end{pmatrix}$$

2. (Linear independence) Let $\{v_1, \dots v_n\}$ be a set of orthonormal vectors in $\mathbb{R}^n$.
   (a) Prove that $\{v_1, \dots v_n\}$ is a linearly independent set (make sure you state what it means to be linearly independent!)
   (b) Suppose you want to write $x \in \mathbb{R}^n$ as a linear combination of $\{v_1, \dots v_n\}$. Explain what this means and give a formula for any quantities you introduce (*cough* like expansion coefficients).

3. (Eigenvalues: duplicated or duplicitous?) We have a theorem asserting that if $\mathbf{A}$ has distinct eigenvalues, it is similar to a diagonal matrix. Is the converse true? Prove or give a counterexample.

Eigenvalue Iterations

1. (Accelerating Convergence)
   (a) Explain in words why Aitken's $\triangle^2$ procedure works.
   (b) Starting with the Power Method algorithm in the course notes, modify the algorithm to accelerate convergence using Aitken's $\triangle^2$ procedure.

2. (Other methods) How else could you quickly solve for an eigenvalue?

3. (Householder's method)
   (a) Show that a Householder matrix

$$F = I - 2\frac{vv^t}{v^t v}$$

   is an orthogonal matrix.
   (b) Explain why you might want to apply a Householder transformation to a matrix.
   (c) What if said matrix is not symmetric?

General Review[1]

1. (Return of Iterative solvers) Think about all the iterative solvers for the linear system $\mathbf{Ax} = \mathbf{b}$ and make a table of the following form:

Method Name	big idea	formula	convergence
⋮	⋮	⋮	⋮

   where "big idea" is the general notion of what you're doing (are you splitting the matrix? minimizing a quadratic functional) and convergence is whatever you know about how fast it converges and under what circumstances.

2. My Matlab code has the following line:

   x = inv(A*B)*b

   and it takes a long time to run. Do you have any suggestions?

3. Given a set of data point s$\{x_j, y_j\}$, what does it mean to

   (a) Interpolate with a polynomial

   (b) Least-squares fit with Legendre polynomials

   (c) Interpolate with trig functions

   (d) Least-squares fit with trig functions

4. What are Chebyshev polynomials and what are your top 3 favorite things about them?

5. How many operations does it take to do a FFT on m data points?

Prepared by ANNA LIEB. Comments and questions: **lieb@math.berkeley.edu**

[1]Disclaimer: this is not meant to be exhaustive, nor is it in any way guaranteed to be related to the actual exam (which the GSI does not write) It is my opinion of concepts you should probably be thinking about, but there are plenty more!