MATH 1B: CALCULUS DISCUSSION SECTION 2: REVIEW QUESTIONS FOR SECOND MIDTERM

Sequences

1. (Quiz 4) Determine if the sequence defined by $a_n = \sin(\frac{2n\pi+\sqrt{7}}{2n})$ converges.
2. (WS 5) True or False?
   (a) If $\lim_{n\to\infty}(a_n + b_n) = L$, then there are numbers L_1 and L_2 such that $\lim_{n\to\infty} a_n = L_1$ and $\lim_{n\to\infty} b_n = L_2$, where $L_1 + L_2 = L$.
   (b) If $\lim_{x\to\infty} f(x) = L$, then $\lim_{n\to\infty} f(n) = L$.
   (c) If $\lim_{n\to\infty} a_n = L$, then $\lim_{n\to\infty} f(a_n) = f(L)$.
   (d) $\lim_{n\to\infty} \frac{a_n}{b_n} = \frac{\lim_{n\to\infty} a_n}{\lim_{n\to\infty} b_n}$
3. (WS 5) Given an example of a sequence which is...
   (I) ...bounded but not convergent
   (II) ...monotone but not bounded
   (III) ...monotone and bounded but not convergent

General Series (Including but not limited to: Alternating Series, Integral Test, Comparison Tests, Limit Comparison Tests, Ratio and Root Tests)

1. (HW 5) Find the value of c if $\sum_{n=2}^{\infty}(1+c)^{-n} = 2$
2. (HW 5) For what values of b does the series $\sum_{n=1}^{\infty} b^{\ln(n)}$ converge?
3. (Persson '09) Find all x that satisfy the equation
$$\sum_{n=0}^{\infty}(-1)^n(n+1)x^{2n+2} = \frac{2}{9}$$
4. (Quiz 6) Is it possible for a series with all positive terms to converge conditionally?
5. (HW 6) Show that the series defined by $\sum_{n=1}^{\infty}(-1)^n b_n$, where $b_n = 1/n^2$ for even n and $b_n = 1/n$ for odd n is divergent. Why does the alternating series test fail?
6. (HW 6) For what values of p is the series $\sum_{n=1}^{\infty}(-1)^n/n^p$ absolutely convergent? Conditionally convergent? item (Quiz 6) Is the series $\sum_{n=1}^{\infty}\sin(1/n)/n^{1/6}$ convergent or divergent?
7. Determine if the series $\sum_{n=1}^{\infty} n\sin(1/n)$ converges or diverges.
8. (HW 6) For what values of k does the series $\sum_{n=0}^{\infty}\frac{n^2}{(kn)!}$ converge? How about $\sum_{n=0}^{\infty}\frac{(n^n)2}{(kn)!}$?

Power Series

1. (Worksheet 7) Suppose a series $\sum_{n=0}^{\infty} a_n(-4)^n$ converges. Does $\sum_{n=0}^{\infty} a_n(2)^n$ converge? How about $\sum_{n=0}^{\infty} a_n(4)^n$?
2. (Persson '09) Find the interval of convergence for the series $\sum_{n=1}^{\infty}(-1)^n\frac{x^n}{n\cdot 3^{2n}}$, making sure to check convergence at the endpoints.
3. (HW 8) Write the indefinite integral $\int x\cos(x^3)dx$ as a power series.

4. (Quiz 8) Use Taylor series to evaluate the limit $\lim_{x\to 0}\left(\frac{1-x^4-\cos(\sqrt{2}x^2)}{2x^8}\right)$

5. (HW 8) Use Taylor's remainder theorem to estimate the $e^{0.1}$ to within 0.00001.

6. Estimate the integral $\int_0^{.5} 1/(x+3)^{1/3}$ correct to five decimal places.

7. (Persson '09) Show that the series

$$y = \sum_{n=0}^{\infty} \frac{x^{2n+1}}{1\cdot 2\cdots(2n+1)}$$

is a solution of the differential equation $y' = 1 + xy$.

8. (Persson '10) Find the Maclaurin series for $f(x) = e^{-x^2}$ and evaluate $f^{(99)}(0)$ and $f^{(100)}(0)$.

9. (HW 8) Find a Maclaurin series for $\sin^{-1}(x)$. (Hint: use the binomial theorem for an appropriate function, and integrate...)

10. Use division of power series to find the first three terms of the Maclaurin series for $\cot(x)$.

Differential Equations

1. (HW 8) Draw a direction field for the differential equation $y' = 1 - y$. Solve the differential equation for $y(1)$ with the initial condition $y(0) = 5$ using (a) Euler's Method and (b) separation of variables.

2. Newton's Law of Cooling says that the rate at which an object cools is proportional to the temperature difference between the object and the temperature of its surroundings. Write a differential equation to describe the temperature T of an object in a room at temperature T_0 and solve it.

3. Solve the differential equation

$$\frac{dy}{dt} = \frac{te^t}{y\sqrt{1+y^2}}$$

4. Suppose a population is modeled by the differential equation

$$\frac{dP}{dt} = kP\left(1 - \frac{P}{K}\right)$$

What is the carrying capacity? Draw a direction field and solve the differential equation using.

WRITE YOUR OWN REVIEW QUESTIONS HERE:

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