MATH 1B: CALCULUS DISCUSSION SECTION 2: QUIZ 7

Please write neatly and show all your work.

1. For the following series, find the values of x for which it converges. Then decide if it is a power series or not

   (a) (2 points)

$$\sum_{n=0} \frac{(-1)^{n+1}\ln(n+1)x^{-2n}}{(4n)!}$$

$$\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty}\left|\frac{x^{-2n-2}\ln(n+2)\,(4n)!}{x^{-2n}\ln(n+1)\,(4n+4)!}\right|$$

$$= \lim_{n\to\infty}\left| x^{-2}\frac{\ln(n+2)}{\ln(n+1)}\frac{\cancel{(4n)!}}{(4n+4)(4n+3)(4n+2)(4n+1)\cancel{(4n)!}}\right|$$

$$= \lim_{n\to\infty}\left|\frac{\ln(n+2)}{\ln(n+1)(4n+4)(4n+3)(4n+2)(4n+1)}\right| |x^{-2}|$$

$= 0$ for all x.

but NOT a power series (has x raised to NEGATIVE POWERS)

   (b) (2 points)

$$\sum_{n=0} \frac{n!(x-4)^{2n+1}}{2^n}$$

$$\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty}\left|\frac{(n+1)!\,(x-4)^{2n+2}}{2^{n+1}}\cdot\frac{2^n}{n!\,(x-4)^{2n+1}}\right|$$

$$= \lim_{n\to\infty} (2)|(x-4)|\,|(n+1)| = \infty$$

unless $x=4$. diverges unless $x=4$.

is a power series.

2. (2 points) Find a power series for

$$\ln(\pi - x)$$

   What is the radius of convergence?

$$\ln(\pi - x) = -\int_0^x \frac{1}{\pi - t}\,dt + \ln(\pi)$$

$$= -\int_0^x \frac{1}{\pi(1-(\frac{t}{\pi}))}\,dt + \ln(\pi)$$

$$= -\frac{1}{\pi}\int_0^x \sum_{n=0}^{\infty}\left(\frac{t}{\pi}\right)^n dt + \ln(\pi)$$

$$= -\frac{1}{\pi}\sum_{n=0}^{\infty}\frac{\pi}{n+1}\left(\frac{x}{\pi}\right)^{n+1} + \ln(\pi)$$

converges when $|x| < \pi$ $R = \pi$

CONTINUED ON OPPOSITE SIDE!

3. (a) (1 points) Re-write

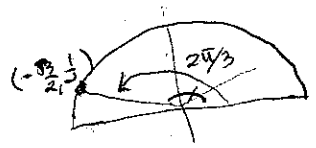
$$\int_0^R \frac{dx}{1+\tan^2(x)}$$

by expressing the integrand as an infinite series.

$$= \int_0^R \sum_{k=0}^{\infty} (-\tan^2(x))^k \, dx$$

(b) (2 points) Now evaluate the integral for $R = \pi/3$ (hint: use a trig substitution).

$$1+\tan^2(x) = \sec^2(x) = \frac{1}{\cos^2(x)}$$



$$\int_0^R \cos^2 x \, dx = \frac{1}{2}\int_0^R 1+\cos(2x)\, dx$$
$$= \frac{1}{2}\left(x + \frac{1}{2}\sin(2x)\right)\Big|_0^R$$
$$= \frac{R}{2} + \frac{1}{4}\sin(2R)$$
$$= \frac{\pi}{6} + \frac{1}{4}\sin\left(\frac{2\pi}{3}\right) = \frac{\pi}{6} + \frac{1}{8}$$

(c) (1 point) If you knew how to integrate each term in your expression in (a), do you expect that it would agree with your answer for (b)?

Nope .... not a power series ... no convergence gauranteed!

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