MATH 1B: CALCULUS DISCUSSION SECTION 2: QUIZ 6

Please write neatly and show all your work.

1. (a) (2 point) If $a_n \geq 0$ for all n, is it possible for $\sum_{n=1}^{\infty} a_n$ to converge conditionally? Why or why not?

No, because if $a_n \geq 0$, then $|a_n| = a_n$ so $\sum_1 |a_n|$ converges if $\sum_1 a_n$ does

(b) (1 ~~points~~ point) Give an example of a series that converges conditionally.

e.g.: $\sum_{n=1}^{\infty} (-1)^n \frac{1}{n}$

2. (3 points) For what values of x does the series

$$\sum_{n=1}^{\infty} \frac{(n+1)x^n}{n!} (-1)^{n+1}$$

converge? Justify your answer.

ratio test: $\dfrac{\left| \dfrac{(n+2)(-1)^{(n+2)} x^{n+1}}{(n+1)!} \right|}{\left| \dfrac{n+1 \, (-1)^{n+1} x^{n+1}}{n!} \right|} = \dfrac{|a_{n+1}|}{|a_n|}$

$$= \left(\frac{(n+2)|x^{n+1}|}{(n+1)n!} \right) \left(\frac{n!}{(n+1)|x^{n+1}|} \right)$$

$$= \frac{n+2}{(n+1)^2} \left| \frac{x^n}{x^{n+1}} \right|$$

$$\frac{|a_{n+1}|}{|a_n|} = \frac{n+2}{(n+1)^2} |x|$$

so: $\lim_{n\to\infty} \left| \frac{a_{n+1}}{a_n} \right| = \lim_{n\to\infty} \frac{n+2}{(n+1)^2} |x| = \lim_{n\to\infty} \frac{\frac{1}{n} + \frac{2}{n^2}}{(1+\frac{1}{n})^2} |x|$

$$= \frac{\lim_{n\to\infty} (\frac{1}{n} + \frac{2}{n^2})}{\lim_{n\to\infty} (1+\frac{1}{n})^2} |x| = 0 \quad \text{for all } x.$$

converges for all x by ratio test.

3. (2 points) Is the series

$$\sum_{n=1}^{\infty}\left(\frac{n^2+3n}{2n^2+5}\right)^n$$

Convergent? Justify your answer.

n^{th} root test $\qquad \sqrt[n]{a_n} = \left(\frac{n^2+3n}{2n^2+5}\right)$

$$\lim_{n\to\infty}\sqrt[n]{a_n} = \lim_{n\to\infty}\frac{n^2+3n}{2n^2+5}$$

$$= \lim_{n\to\infty}\frac{1+\frac{3}{n}}{2+\frac{5}{n^2}} = \frac{\lim_{n\to\infty}(1+\frac{3}{n})}{\lim_{n\to\infty}(2+\frac{5}{n^2})}$$

$$= \frac{1}{2}$$

$\lim_{n\to\infty}\sqrt[n]{a_n} = \frac{1}{2} < 1 \Rightarrow \sum_1^{\infty} a_n$ converges by n^{th} root test.

4. (2 points) Is the series

$$\sum_{n=1}^{\infty}\frac{\sin\left(\frac{1}{n}\right)}{n^{1/6}}$$

Convergent? Justify your answer.

Limit Comparison with $\sum_1 \frac{1}{n^{7/6}} = \sum_1 b_n$

$\frac{7}{6} > 1 \Rightarrow b_n$ convergent.

Now examine *

$$\lim_{n\to\infty}\frac{\frac{\sin(\frac{1}{n})}{n^{1/6}}}{\frac{1}{n^{7/6}}} = \lim_{n\to\infty} n\sin(\tfrac{1}{n}) = \lim_{n\to\infty}\frac{\sin(\frac{1}{n})}{\frac{1}{n}}$$

Let $\frac{1}{n} = x$. Equivalent limit is $\lim_{x\to 0}\frac{\sin x}{x} \overset{\text{L'Hopital's Rule}}{=} \lim_{x\to 0}\frac{\cos x}{1} = 1$

So since $\lim_{n\to\infty}\frac{\frac{\sin(\frac{1}{n})}{n^{1/6}}}{b_n} = 1$ and $\sum_1 b_n$ is convergent,

this series also converges.

* can also do L'Hopital's rule for function $\frac{\sin(\frac{1}{x})}{\frac{1}{x}}$ as $x\to\infty$.

Prepared by ANNA LIEB. Comments and questions: lieb@math.berkeley.edu