MATH 128B: QUIZ 4

4 points

We want to solve a nonlinear equation $\mathbf{F}(\mathbf{x}) = 0$. We have a method to generate a sequence of approximate solutions: $\mathbf{x}^{(0)}, \mathbf{x}^{(1)}, \mathbf{x}^{(2)}, \ldots$. Denote the error at the k^{th} step via

$$e^k = ||\mathbf{x}^{(k)} - \mathbf{x}||_\infty$$

where $\mathbf{x}$ is the true solution.

1. (Convergence) Write down a relationship satisfied by e^k and e^{k-1} given that the method converges
   (a) linearly
   (b) quadratically
   Make sure do define any symbols you use.

(a) $e^k \leq C e^{k-1}$ for some constant C
(C depends, potentially, on # dimensions, bounds of derivatives of F, etc..)

(b) $e^k \leq C (e^{k-1})^2$ for some constant C

Note these relations are inequalities!

2. Give an example (just the name, not the formula) of a nonlinear solution method that converges
   (a) linearly Fixed Point, Steepest Descent
   (b) quadratically Newton's

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3. (Newton's Method) Write down the formula for an update step using Newton's Method, defining any symbols that you use.

$$x^{k+1} = x^k - J^{-1}(x^k)\,F(x^k)$$

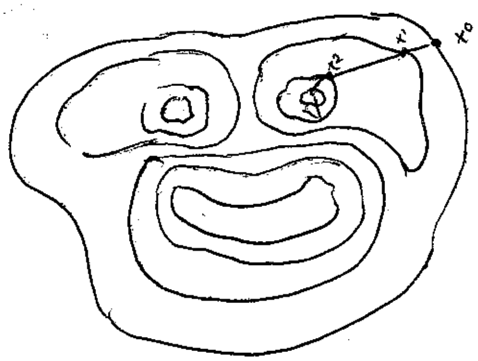
$J(x^k)$ is Jacobian for F

$$(J)_{ij} = \frac{\partial F_i}{\partial x_j}$$

4. Name a possible drawback of Newton's Method and how you would address it.

- have to evaluate/invert J at each step
  - ⇒ could use Sherman-Morrison formula to update J
- have to analytically solve for J
  - ⇒ could use finite difference approx.
- can diverge if initial guess is far off
  - ⇒ could use another method to get "close"
- Jacobian could be singular at the minimum
  - ⇒ just use another method!

5. (Bonus!) Draw a self-portrait using level curves of a nonlinear function and the path of a steepest descent algorithm.



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