1. A combinatorics trick in calculus! Use a counting argument to show that the product rule for the n^{th} derivative is

$$\left(\frac{d}{dx}\right)^n (f \cdot g) = \sum_{k=0}^{n} \binom{n}{k} \left(\frac{d}{dx}\right)^k (f) \cdot \left(\frac{d}{dx}\right)^{n-k} (g)$$

2. List three ways to show a combinatorial identity is true. Now show that

   (a)
$$\sum_{k=0}^{n} \binom{n}{k} 5^k = 6^n$$

   (b)
$$\binom{n}{n-k} = \binom{n}{k}$$

   (c)
$$\sum_{k=0}^{n} k(k-1)(k-2)\binom{n}{k} = n(n-1)(n-2)2^{n-3} \quad \text{(assume } n > 2\text{)}$$

3. (a) Let A be the event that you have a bit strings of length 6 which contains at least 2 zeros. Find the number of ways that A can occur by adding together all the possible cases.

   (b) Use the fact that $\sum_{k=0}^{n} \binom{n}{k} = 2^n$ to come up with a simple expression for your answer above.

   (c) What is the complement of event A? Count the number of ways it can happen and subtract from the total number of ways to make a bit string of length 6.

   (d) It seems like we didn't need the inclusion-exclusion principle to do part (c)? Why is that?

4. (a) Let B be the event that you have a four letter word that has at least one 'g' or 'h'. Find the number of ways B can happen.

   (b) Why do we need the inclusion-exlusion principle now?