

Group 2, Presentation 2 (Fri. Nov 4th)

Let $M = \{(x, y, z) \mid x^2 + y^2 = z^2 + 1\}$ be the hyperboloid.

Let $\gamma(t) = (\gamma_x(t), \gamma_y(t), \gamma_z(t)) = (\sqrt{1+t^2} \cos(t), \sqrt{1+t^2} \sin(t), t)$
(this is a curve on M)

i) Let $p = (p_x, p_y, p_z) \in M$

We can picture $T_p M$ as follows:

$$\text{Let } f(x, y, z) = x^2 + y^2 - z^2 - 1$$

$$\text{if } a_p = \left. \frac{\partial f}{\partial x} \right|_p, b_p = \left. \frac{\partial f}{\partial y} \right|_p, c_p = \left. \frac{\partial f}{\partial z} \right|_p$$

Then we can identify $T_p M$ with the plane

$$T_p = \{(x, y, z) \mid a_p(x - p_x) + b_p(y - p_y) + c_p(z - p_z) = 0\}$$

Compute a_p, b_p, c_p & plot M & T_p

$$f_{\text{or}}: p = (-\sqrt{1+\pi^2}, 0, \pi) \in M$$

$$\text{if } p = (1, 0, 0) \in M.$$

ii) Plot $\gamma(t)$, and the lines

$$l_s = \left\{ \left(\gamma_x(t) + s \frac{\partial \gamma_x(t)}{\partial t}, \gamma_y(t) + s \frac{\partial \gamma_y(t)}{\partial t}, \gamma_z(t) + s \frac{\partial \gamma_z(t)}{\partial t} \right) \mid s \in \mathbb{R} \right\}$$

at $t=0, t=\frac{\pi}{2}, t=\pi$. Are any of $l_0, l_{\frac{\pi}{2}}, l_\pi$ subsets of the planes $T_{(1, 0, 0)}$ or $T_{(-\sqrt{1+\pi^2}, 0, \pi)}$?