

# Group 1, Presentation 2 (Thurs, Nov 4<sup>th</sup>)

Let  $S^2 = \{(x, y, z) \mid x^2 + y^2 + z^2 = 1\}$  be the sphere.

Consider the loxodrome curve (with constant angle  $\beta$  to all meridians)

$$\gamma(t) = (\cos t \cdot \operatorname{sech}(t \cdot \cot(\beta)), \sin(t) \operatorname{sech}(t \cdot \cot(\beta)), \tanh(t \cdot \cot(\beta)))$$

- i) Show that  $\gamma(t) \in S^2 \quad \forall t$  (i.e.  $(\gamma'(t))^2 + (\gamma''(t))^2 + (\gamma''(t))^2 = 1$ )
- ii) Show that  $\gamma(0) = (1, 0, 0)$

Consider the curve

$$\tau(t) = (\cos(t \csc \beta), \sin(\beta) \sin(t \csc \beta), \cos(\beta) \sin(t \csc \beta))$$

- iii) Show that  $\tau(t) \in S^2 \quad \forall t$
- iv) Show that  $\tau(0) = (1, 0, 0)$
- v) Show that  $\gamma \& \tau$  are equivalent curves (to first order)  
at  $(1, 0, 0) \in M$ , i.e. for any  $f \in C^\infty(S^2 M)$

$$\frac{\partial f \circ \gamma}{\partial t} \Big|_{t=0} = \frac{\partial f \circ \tau}{\partial t} \Big|_{t=0}$$

- vi) Plot  $\gamma \& \tau$ .