

Group O Presentation 2 (Wed. Nov 2nd)

Let M be the set

$$M = \{(x, y, z) \in \mathbb{R}^3 \mid x^2 + y^2 = z^2 + 1\}$$

i) Show that M is a manifold (use the regular value theorem for the function $f(x, y, z) = x^2 + y^2 - z^2 - 1$).
What does M look like?

ii) let $U = M \setminus \{(0, \sqrt{1-z^2}, z) \mid z \in \mathbb{R}\}$

& $\varphi: U \rightarrow \mathbb{R}^2$ be given by $\varphi(x, y, z) = \left(\frac{x}{\sqrt{1+z^2-y}}, z \right)$

Similarly, let $V = M \setminus \{(0, -\sqrt{1-z^2}, z) \mid z \in \mathbb{R}\}$

$\psi: V \rightarrow \mathbb{R}^2$ be given by $\psi(x, y, z) = \left(\frac{x}{y+\sqrt{1+z^2}}, z \right)$

Compute the transition maps $\varphi \circ \psi^{-1}$ & $\psi \circ \varphi^{-1}$.
Show that (U, φ) & (V, ψ) are compatible.

iii) Let $\gamma: \mathbb{R} \rightarrow M$ be defined in local coordinates
by $\varphi \circ \gamma(t) = \left(\frac{\cos(t)}{1 - \sin(t)}, t \right)$ for $t \notin \left\{ \frac{\pi}{2} + 2k\pi \mid k \in \mathbb{Z} \right\}$

$$\psi \circ \gamma(t) = \left(\frac{\cos(t)}{1 + \sin(t)}, t \right) \text{ for } t \notin \left\{ -\frac{\pi}{2} + 2k\pi \mid k \in \mathbb{Z} \right\}$$

Show that $\gamma: \mathbb{R} \rightarrow M$ is smooth

iv) Let $i: M \rightarrow \mathbb{R}^3$ be the inclusion
 $i(x, y, z) = (x, y, z)$

Compute $i \circ \gamma: \mathbb{R} \rightarrow \mathbb{R}^3$ What does it look like?