

Group 1 Presentation (Friday Oct 28<sup>th</sup>)

Problem Show that the map

$$F: \mathbb{R}\mathbb{P}^n \longrightarrow \text{Mat}_{(n+1) \times (n+1)} \cong \mathbb{R}^{(n+1) \times (n+1)}$$

defined by  $F([x^0 : \dots : x^n]) = \frac{1}{(x^0)^2 + \dots + (x^n)^2} \begin{pmatrix} x^0 x^0 & x^0 x^1 & \dots & x^0 x^n \\ x^1 x^0 & x^1 x^1 & \dots & x^1 x^n \\ \vdots & \vdots & \ddots & \vdots \\ x^n x^0 & x^n x^1 & \dots & x^n x^n \end{pmatrix}$

is continuous (smooth, in fact) and injective

Conclude that for any pair of distinct points

$$p = [p^0 : \dots : p^n], q = [q^0 : \dots : q^n] \in \mathbb{R}\mathbb{P}^n \quad \exists \text{ indices } 0 \leq i, j \leq n$$

s.t.

$$F_{i,j}(p) \neq F_{i,j}(q), \text{ where } F_{i,j}([x^0 : \dots : x^n]) = \frac{x^i x^j}{(x^0)^2 + \dots + (x^n)^2}.$$

In particular,  $\mathbb{R}\mathbb{P}^n$  is hausdorff