Integration of Poisson manifolds and canonical relations:

w. A. Cattaneo (arXiv: 1401.7319).

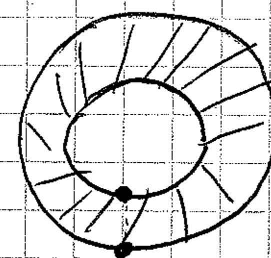
Idea: Advertise an integration procedure via canonical relations assoc. to T^*PM. (M, π) Poisson; it exists for non-integrable Poisson mfds. (tho it does not satisfy Crainic-Fernandes conditions).

Plan
1) Integration of T^*M (M, π) Poisson
2) Relational symplectic groupoid. (construction and relationship with M)
3) Open questions (reln. with Top field theories)

1). Lie III: Given a finite dimensional Lie algebra $\mathfrak{g}$, $\exists!$ (simply connected) group G s.t. $T_eG \overset{l.a}{\simeq} \mathfrak{g}$.

Proof: ~~Path space construction~~:

Reduction argument:



$$P\mathfrak{g} := \Omega^1(S^1, \mathfrak{g})$$

Connections in ~~$\#$~~ $\mathfrak{g} \times S^1 \longrightarrow S^1$.

$$\gamma_1 \sim \gamma_2 \text{ if } \exists \ \nabla \in \Omega^1_{flat}(An, \mathfrak{g}) : \quad \nabla|_{in} = \gamma_1, \quad \nabla|_{out} = \gamma_2.$$

- ~~Fixing a point gives the unit~~.
- Group structure (Loop concatenation).

Unit:  γ $\quad G = $ Flat connections / gauge trans f.

This argument can be 'extended' to Lie algebroids

$\mathfrak{g} \longmapsto A \to M$

$G \longmapsto G \rightrightarrows M$

$A = T^*M \qquad (M, \pi)$

$\pi^\#: T^*M \to TM \ ; \qquad [\![d\alpha, d\beta]\!] = d\{f,g\}$ + Leibniz extended.

$G \rightrightarrows M$ is symplectic. $\qquad G {}_s\!\times_t G \xrightarrow{\mu} G \underset{t}{\overset{s}{\rightrightarrows}} M$, ι

(G, ω) is a sympl. f.d manifold.

$gr\,\mu \subset G \times G \times \overline{G}$ is Lagrangian.

Crucial point: • G not always exists as a smooth mfd.

(depends on the variation of periods associated to the symplectic volume of the symplectic foliation of T^*M. $\quad G = T^*PM \big/ \sim_{\text{gauge equiv.}}$

3 dim Ex

v.b. morphisms btw $TI \to T^*M$.

$\sim$ = ~~gauge~~ Lie algebroid homotopy

• Groupoid information is encoded in canonical relations (Lag. submanifolds) and coisotropic rels (cois. subm. of Poisson mfds)

graph $\mu \subset G \times G \times \overline{G}$ is cano. rel.

graph $\varepsilon \subset G$ " "

graph $s \subset G \times \overline{M}$ is coisotropic.

" " $t \subset G \times M$.

~~Pr~~ Proposal: Describe G before reduction.

Set-up: Poisson sigma model

Poisson sigma model (PSM)

Σ, (M, π)

surface

$\partial\Sigma$.

$\Phi = \{ \text{v.b. morphisms btw } T\Sigma \longrightarrow T^*M \}$.

$\varphi = (X, \eta) \quad X: \Sigma \longrightarrow M \in \mathrm{Map}^k(\Sigma, M)$

$\eta \in \Gamma^k(\mathrm{Hom}(T\Sigma, X^*(T^*M)))$

$$S(\varphi) = \int_\Sigma \eta_i \wedge dX^i + \tfrac{1}{2}\pi^{ij}(X)\eta_i \wedge \eta_j$$

$$dX^i = X^*(dx^i)$$

Boundary fields

$\Phi_\partial = \{ \text{v.b. morph btw } T[0,1] \longrightarrow T^*M \}$

C_π: Euler-Lagrange $(\Phi)_\partial$ = Lie algebroid morph $T[0,1] \longrightarrow T^*M$.

Theo (Cattaneo-Felder) $\cdot$ C_π is coisotropic $\subset T^*PM$

If smooth, $\underline{C_\pi} = (G, \omega) \rightrightarrows (M, \pi)$

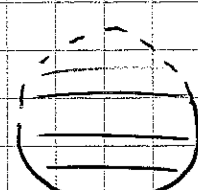
Proposal: Describe (G, ω) before reduction

- Infinite-dimensional Banach submfds
- Groupoid axioms hold 'up to equivalence'

L_1, L_2, L_3.

L_1: $pt \longrightarrow\!\!\!\!\!\!\!/\;\; T^*M$

$\{ \varphi \in T^*PM : \varphi \overset{g.e}{\sim} ct \}$

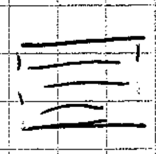


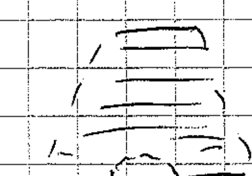
L_3: $(\gamma, \gamma', \gamma'') \in T^*PM \times T^*PM \longrightarrow\!\!\!\!\!\!\!/\;\; T^*PM$

$\gamma * \gamma' \overset{g.e}{\sim} \gamma''$

L_2: $T^*M \longrightarrow\!\!\!\!\!\!\!/\;\; T^*M$

$\{ \gamma, \tilde{\gamma} \}$: $\gamma \overset{g.e}{\sim} \tilde{\gamma}$





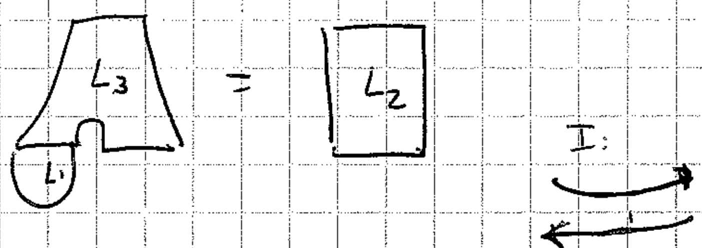
Theo (C-C). In the smooth case

$$(G,\omega) = C_\pi / L_2 , \quad \mathcal{E} = L_1/L_2$$
$$\mu = L_3/L_2 .$$

<u>Theo</u> (C-C) In general. L_1, L_2, L_3 satisfy algebraic relations

- $L_3 \circ (L_3 \times id) = L_3 \circ (id \times L_3)$. (relational symplectic gpd $(\mathcal{G}, L, I)$

$$\begin{cases} \mathcal{G} = T^*PM \\ L = I \circ L_3 \end{cases}$$

- 

and it always exists.

∃! Poisson structure

<u>Theo</u> Any regular relational symplectic groupoid ($M = L_1/L_2$ is fin. dim)

then $s: C \longrightarrow M$ is a forward Dirac map.

$C = L_2 \circ \mathcal{G}$.

<u>Open Questions</u> :
- Integration with genus
- How locality plays role in the groupoid structure.
- Derived formulation of PSM (extended AKSZ theories).

⋮