Worksheet 24

1 Z-Test

1. When Thanos snapped his fingers, everyone had a $p = 0.5$ chance of disintegrating. I think that this probability was much lower for the original Avengers. Out of the 6 of them, no one got disintegrated. Can you reject the null hypothesis that there was a $p = 0.5$ chance of each of them disintegrating with an $\alpha = 0.05$?

   Solution: Our null hypothesis is H0 : $p = 0.5$ and alternate hypothesis H1 : $p < 0.5$. We want the probability of at least that unlikely of a scenario which is $P(X \leq 0)$ less than or equal to 0 of them got disintegrated. This is a binomial distribution so the probability of this happening is $P(X \leq 0) = P(X = 0) = \frac{1}{2^6} = 0.015 < \alpha$. Therefore we can reject the null hypothesis and say that they were given some special treatment.

2. An infomercial claims that a miracle drug will cause you to grow all your hair back. There are 36 brave participants and surprisingly 10 people regrew their hair. If normally 10% of people regrow their hair, can you say that this drug worked?

   Solution: We would expect that 10% of people will regrow their hair with standard deviation $= \sigma = \sqrt{p(1-p)} = \sqrt{0.1 \times 0.9} = 0.3$. The central limit theorem says that with a sample of 36 people, we expect that 10% of people regrow their hair with a standard deviation of $\sigma/\sqrt{n} = \frac{0.3}{\sqrt{36}} = 0.05$. There are $10/36 \approx 28\%$ who regrew their hair. The z score is $z(|0.28 - 0.1|/0.05) \approx Z(3.56) < \alpha$. Therefore, we can reject the null hypothesis and say that this drug does help you grow your hair back.

3. You flip a coin 100 times and get 55 heads. Can you say that it is biased towards heads? (use $\alpha = 0.05$)

   Solution: The null hypothesis is that the coin is unbiased and hence $p = 0.5$. The standard deviation $= \sigma = \sqrt{p(1-p)} = 0.5$. Thus, the central limit theorem tells us that the percentage of coin flips we get is approximately normally distributed with a standard deviation of $\sigma/\sqrt{n} = \frac{0.5}{\sqrt{100}} = 0.05$. There are $55/100 = 55\%$ of heads. The z score is $z(|0.55 - 0.5|/0.05) = Z(1) > \alpha$. Therefore, we cannot reject the null hypothesis and say that this drug does help you grow your hair back.

4. An infomercial claims that a miracle drug will cause you to grow all your hair back. There are 100 brave participants and this time 20 people regrew their hair. If normally 10% of people regrow their hair, can you say that this drug worked?

   Solution: We would expect that 10% of people will regrow their hair with standard deviation $= \sigma = \sqrt{p(1-p)} = \sqrt{0.1 \times 0.9} = 0.3$. The central limit theorem says that with a sample of 100 people, we expect that 10% of people regrow their hair with a standard deviation of $\sigma/\sqrt{n} = \frac{0.3}{\sqrt{100}} = 0.03$. There are $20/100 = 20\%$ who regrew their hair. The z score is $z(|0.2 - 0.1|/0.03) = Z(3.33) < \alpha$. Therefore, we can reject the null hypothesis and say that this drug does help you grow your hair back.

2 T-Test

Concept:What is the t-statistic, and what is it used for?

For a sample of size n, the t-statistic is a measure of how far the sample mean Y_n lies from the hypothesized population mean μ_0, measured in units of the standard error in the mean $s/\sqrt{n}$. The t-statistic is given by

$$T_{n-1} = \frac{Y_n - \mu_0}{s/\sqrt{n}}$$

It is used during hypothesis testing to determine whether the sample data are compatible with the null hypothesis. It usually deal with the case that has small sample size.

1. The heart rates of 20 patients in an ICU have mean 95.3beats/min and standard deviation 16.9 beats/min. Are heart rates from ICU patients unusual given normal heart rate has mean of 72 beats/min?

   (a) What is the degree of freedom?

   Solution: degree of freedom $= n - 1 = 20 - 1 = 19$

   (b) What is the t-statistic?

   Solution: $T_{n-1} = \frac{95.3-72}{16.9/\sqrt{20}} \approx 6.17$

2. Scores on the SAT math section follow a normal distribution with mean 500. You suspect you are better than the average SAT taker at math and so take five different SAT math tests. If your scores on these exams are 570, 620, 710, 440, and 710:

   (a) calculate the average and sample standard deviation of your scores. **Solution:** average = 610, $s \approx$

   (b) calculate the t-statistic of your scores. **Solution:** $t = \frac{610-500}{112/\sqrt{5}} \approx 2.2$

   (c) get a p-value for this result. At $\alpha = 0.05$ significance, should you conclude that you're good at SAT math tests? **Solution:** $p = 0.046$, we should reject the null hypothesis and conclu SAT math tests

   (d) Suppose your two 710 scores had actually been perfect 800s. How does your t-statistic change, and why? **Solution:** average = 646, $s = 155.18$, $t \approx 2.1$, so our p-value actually goes u s went up by more than the average