

Kap: A Tale of Two Cities

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Through Professor Hyman Bass, Kap was my mathematical grandfather. This and the fact that Kap offered me my first job as an Instructor at Chicago were perhaps not statistically independent events. The time was forty years ago, when Kap was finishing his 5-year term as Chicago Chair. The offer was consummated by a Western Union telegram — the 1960s' equivalent of email. Kap didn't ask for my C.V. (I wouldn't have known what that was); nor did he want to know my “teaching philosophy” (I had none). For my annual salary, Kap offered me \$8000 — a princely sum compared to my then T.A. stipend of \$2000 at Columbia University. I have joked to my colleagues that I'll always remember Kap as the only person through my whole career to have ever quadrupled my salary. But in truth, a ticket to Chicago's famed Eckhart Hall for postdoctoral studies was more than anything a fledgling algebraist could have dreamed. For this wonderful postdoctoral experience Kap afforded me through his unconditional confidence in a mathematical grandson, I have always remained grateful.

I met Kap for the first time in the Fall of 1967 when I reported to work in Hyde Park. By that time, Kap had already taught for 22 years at the University of Chicago. Although he was Canadian by birth, Chicago had long been his adopted home and workplace: it is, appropriately, the city where our “tale” begins.

For students interested in abstract algebra, Kaplansky is virtually a household name. In graduate school, I first learned with great delight Kap's marvelous theorem on the decomposition of projective modules, and his surprisingly efficient treatment of homological dimensions, regular local rings and UFDs. It was to take me forty more years, however, to get a fuller glimpse of the breadth and depth of Kap's total mathematical output. In these days of increasing specializations in mathematics, we can only look back in awe to Kap's trailblazing work through an amazingly diverse array of research topics, ranging from valuation theory, topological algebra, continuous geometry, operator algebras and functional analysis, to modules and abelian groups, commutative and homological algebra, P.I. rings and general noncommutative rings, infinite-dimensional Lie algebras, Lie superalgebras (supersymmetries), as well as the theory of quadratic forms in both its algebraic and arithmetic flavors. Kap was master of them all. In between the “bigger” works, Kap's publications also sparkled with an assortment of shorter but very elegant notes, in number theory, linear algebra, combinatorics, statistics and game theory. All of this, still, did not include the many other works recorded in “fourteen loose-leaf notebooks” (referred to in the preface of [1]) that Kap had kept for himself over the years. One cannot help but wonder how many more mathematical gems have remained hidden in those unpublished notebooks!

For me, reading one of Kap's papers has always proved to be a richly rewarding experience. There are no messy formulas or long-winded proofs; instead, the reader is treated to a smooth flow of novel mathematical ideas carefully crafted to perfection by an artisan's hand. Some authors dazzled us with their technical brilliance; Kap won you over by the pure soundness of his mathematical thought. In his publications, Kap was much more given to building new conceptual and structural frameworks, than going down single-mindedly into a path of topical specialization. This style of doing mathematics made him a direct intellectual descendant of Emmy Noether and John von Neumann. As a consequence, many of Kap's mathematical discoveries are of a fundamental nature and a broad appeal. The famous Kaplansky Density Theorem for unit balls, and his important inaugural finiteness result in the theory of rings with polynomial identities are only two of the most outstanding examples.

Those of us who have had the privilege of listening to Kap all knew that he was extremely well spoken and had indeed a very special way with words. However, this gift did not always manifest itself when Kap was in social company with Chellie. It was quite clear to all his colleagues who Kap thought was the better orator in the family. Dinner parties the Kaps attended were often replete with Chellie's amusing stories about the Chicago Department and its many colorful mathematical personalities, from an austere André Weil down to the more transient, sometimes bungling graduate students over the years. As Chellie recounted such funny stories with her characteristic zest and candor, Kap would listen admiringly on the side – without interruption. Only at the end of a story would he sometimes add a clarifying comment, perhaps prompted by his innate sense of mathematical precision, such as “Oh, that was 1957 summer, not fall.”

Kap's extraordinary gift in oral (and written) expression was to find its perfect outlet in his teaching, in which it became Chellie's turn to play a supporting role. In the many lecture courses Kap gave at the University of Chicago in a span of 39 years, he introduced generation after generation of students to higher algebra and analysis. In those courses he taught that were of an experimental nature, Kap often directly inspired his students to new avenues of investigation, and even to original mathematical discoveries at an early stage. (Schanuel's Lemma on projective resolutions, proved by Stephen Schanuel in Kap's Fall 1958 Chicago course in homological algebra, was perhaps the best known example.) It was thus no accident that Chicago graduate students flocked to Kap for theses supervision. Over the years, Kap directed doctoral dissertations in almost every one of the mathematical fields in which he himself had worked. Many of Kap's 55 Ph.D. students from Chicago are now on the senior faculty at major universities in the U.S. Currently, the Mathematics Genealogy Project listed Kap as having 627 descendants — and counting. This is the *second* highest number of progeny produced by mathematicians in the US who had their own Ph.D. degrees awarded after 1940. We leave it as an exercise for the reader to figure out who took the top honor in that category, with the not-too-useful hint that this mathematician was born a year after Kap.

While Kap had clearly exerted a tremendous influence on mathematics through his own research work and that of his many Ph.D. students, the books written by him were a class by themselves. The eleven books listed in the side-bar on this page traversed

the whole spectrum of mathematical exposition, from the advanced to the elementary, reaching down to the introduction of mathematics to non-majors in the college. *Differential Algebra* typified Kap's broad-mindedness in book writing, as its subject matter was not in one of Kap's specialty fields. On the other hand, *Infinite Abelian Groups* introduced countless readers to the simplicity and beauty of a subject dear to Kap's heart, while *Rings of Operators* served as a capstone for his pioneering work on the use of algebraic methods in operator algebras. *Lie Algebras*, *Commutative Rings*, as well as *Fields and Rings*, all originating from Kap's graduate courses, extended his classroom teaching to the mathematical community at large, and provided a staple for the education of many a graduate student worldwide, at a time when few books covering the same materials at the introductory research level were available. In these books, Kap sometimes experimented with rather audacious approaches to his subject matters. For instance, *Commutative Rings* will probably go down on record as the only text in commutative algebra that totally dispensed with any discussion of primary ideals or artinian rings.

As much as his books are appreciated for their valuable and innovative contents, Kap's great fame as an author derived perhaps even more from his very distinctive writing style. There is one common characteristic of Kap's books: they were all *short* — something like 200 pages was the norm. (Even *Selected Papers* [1] had only 257 pages, by his own choice.) Kap wrote mostly in short and simple sentences, but very clearly and with great precision. He never belabored technical issues, and always kept the central ideas in the forefront with an unerring didactic sense. The polished economy of Kap's writing makes it all at once fresh, crisp, and engaging for his readers, while his mastery and insight shone on every page. The occasional witty comments and asides in his books — a famous Kaplansky trademark — are especially a constant source of pleasure for all. In retrospect, Kap was not just a first-rate author; he was truly a superb expositor and a foremost mathematical stylist of his time.

After I moved from Chicago to Berkeley, my contacts with Kap became sadly rather infrequent. So imagine my great surprise and delight, 16 years later, when word first came out that Kap was to retire from the University of Chicago, in order to succeed Chern as the Director of MSRI! In the Spring of 1984, the Kaplanskys arrived and established their new abode a few blocks north of the university campus — in Berkeley, California, the second city of our tale.

The Math Departments at Chicago and Berkeley share much more than the “U.C.” designation of the universities to which they belong. There has been a long (though never cantankerous) history of the Berkeley Department recruiting its faculty from the Chicago community, starting many years ago with Kelley, Spanier, and Chern. Indeed, when Kap himself joined the U.C.B. faculty in 1984, there were at least as many as 16 mathematicians there who had previously been, in one way or another, associated with the University of Chicago. It must have given Kap a tinge of “nostalgia” to be reunited, in such an unexpected way, with so many former graduate students, postdoc's, and colleagues from his beloved Chicago Department. But if anyone had speculated that, by coming West, Kap was to spend his golden years resting on his laurels, he or

she could not have been more wrong. In fact, as soon as Kap arrived at Berkeley in 1984, he was to take on, unprecedentedly, two simultaneous tasks of herculean proportions: (a) to head a major mathematics research institute in the US, and (b) to preside over the largest mathematical society in the world — the AMS.¹

Other contributors to this memorial article are in a much better position than I to comment on Kap’s accomplishments in (a) and (b) above, so I defer to them. In the following, my reminiscences on Kap’s Berkeley years are more of a personal nature. From 1984 on, I certainly had more occasions than ever before to interact mathematically with Kap — discussing with him issues in quadratic forms and ring theory. Kap seemed to favor the written mode of communication (over the oral), but his letters were just as concise as his books. I still have in my prized possession an almost comical sample of Kap’s terseness, in the form of a covering letter for some math notes he sent me. Written out on a standard-size 8" × 11" MSRI letterhead, the letter consisted of 12 words: “Dear Lam: I just did a strange piece of ring theory. Kap.” It was briefly — but of course unambiguously — dated: “Apr. 11 /97”.

Another interaction with Kap in 1998 led to some mathematical output. In preparation of a special volume in honor of Bass’s 65-th birthday, I was very much hoping to commission an article from Kap. In his usual self-effacing fashion, Kap protested that he had really nothing to write about. However, after much persuasion on my part (stressing that he must write for Bass), he gave in and wrote up in his impeccable hand a short note in number theory [2]. Glad that my tactics had paid off, I worked all night to set Kap’s written note in \TeX , and delivered a finished printout to him early the next morning. Kap was surprised; he thanked me profusely, but said that maybe he shouldn’t have written his article. It was too late.

One of Kap’s best known advices to young mathematicians was to “spend some time every day learning something new that is disjoint from the problem on which you are currently working, . . . and read the masters” [3]. Amazingly, even after reaching his 70s, Kap still took his own advice personally and literally. In all the years he was in Berkeley, Kap made it his habit to go to every Monday’s Evans-MSRI talk and every Thursday’s math department colloquium talk. He even had a favorite seat on the left side of the front row in the colloquium room, which, in deference to him, no local Berkeley folks would try to occupy. In the years 1995–97 when I worked in MSRI, I saw Kap quite frequently at the periodicals table in the library, poring over the *Mathematical Reviews* to keep himself abreast with the latest developments in mathematics. And he read the masters too, e.g. in connection with his work on the integral theory of quadratic forms. Members of MSRI have reported sightings of Kap using a small step-ladder in the library to reach a certain big book on a high shelf, and putting the book back in the same fashion after using it (instead of leaving it stray on a table). That tome was an English translation of *Disquisitiones Arithmeticae*: the fact that even a 6-foot tall Kap needed a step-ladder to access it was perhaps still symbolic of the lofty position of the work of the 20-year old Carl Friedrich Gauss.

¹Kaplansky served as AMS President-Elect in 1984, and President for the 2-year period 1985–86.

My 2-years' stay at MSRI was rich with other remembrances about Kap. Undoubtedly, a highlight was Kap's 80-th birthday fest in March, 1997, which was attended by three MSRI Directors and six MSRI Deputy Directors, as well as visiting dignitaries such as Saunders Mac Lane, Tom Lehrer, and Constance Reid. Another most memorable gathering was the Holiday Party in December, 1996, where a relaxed and jovial Kap sang some of his signature songs for us all, accompanying himself on the piano in the MSRI atrium. His energetic, sometimes foot-stomping performance really brought down the house! It saddens me so much to think that, now that Kap is no longer with us, these heart-warming events will never be repeated again.

Twenty years may have been only about a third of Kap's professional life, but I hope that Kap cherished his twenty years in Berkeley with as much fondness as he had cherished his 39 years in Chicago. Those were the two cities (and universities) of his choice, for a long and very distinguished career in mathematics. In Chicago, Kap was a researcher, a chairman, a teacher, a mentor, and an author. In Berkeley, while remaining a steadfast researcher, Kap also became a scientific leader, a senior statesman, and a universal role model. In *each* of these roles, Kap served his profession with devotion, vigor, wisdom, and unsurpassed insight. His lifetime work has profoundly impacted 20th century mathematics, and constituted for us an amazingly rich legacy.

On a personal level, Kap — mathematical grandpa and algebraist *par excellence* — will continue to occupy a special place in my heart. I shall miss his great generosity and easy grace, but thinking of Kap and his towering achievements will always enable me to approach the subject of mathematics with hope and joy.

References

- [1] I. Kaplansky: *Selected Papers and Other Writings*, Springer-Verlag, Berlin-Heidelberg-New York, 1995.
- [2] I. Kaplansky: *A salute to Euler and Dickson on the occasion of Hy's 65-th birthday*, in "Algebra, K -theory, Groups, and Education" (T. Y. Lam, A. Magid, eds.), Contemp. Math. **243** (1999), pp. 79-81, A.M.S., Providence, R.I.
- [3] I. Kaplansky: Response to Steele Prize Citation (Career Award), Notices of the Amer. Math. Soc. **36** (1989), p. 836.

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Books of Irving Kaplansky

- **Infinite Abelian Groups**, 1954, 1969
- **An Introduction to Differential Algebra**, 1957, 1976
- **Introduction to Galois Theory** (in Portuguese), 1958
- **Rings of Operators**, 1968
- **Fields and Rings**, 1969, 1972
- **Linear Algebra and Geometry. A Second Course**, 1969, 1974
- **Commutative Rings**, 1970, 1974
- **Lie Algebras and Locally Compact Groups**, 1971, 1974
- **Set Theory and Metric Spaces**, 1972, 1977
- **Matters Mathematical** (with I. Herstein), 1978
- **Selected Papers and Other Writings**, 1995