Name: _____ Section: _____

Math 54 Lec 006 Quiz 9

Tuesday, July 24, 2018

Justify your assertions; include detailed explanation, and show your work. Closed book exam, no sheet of notes and no calculator. This quiz is worth 9 points total.

1. (3 points) Let $A = \begin{pmatrix} 1 & 3 \\ 1 & -1 \\ 1 & 1 \end{pmatrix}$. Orthogonally diagonalize $A^T A$.

$A^T A = \begin{pmatrix} 3 & 3 \\ 3 & 11 \end{pmatrix}$ which has eigenvalues $\lambda_1 = 12, \lambda_2 = 2$. For $\lambda_1 = 12$, the corresponding eigenvector is $\frac{1}{\sqrt{10}} \begin{pmatrix} 1 \\ 3 \end{pmatrix}$, while for $\lambda_2 = 2$, the corresponding eigenvector is $\frac{1}{\sqrt{10}} \begin{pmatrix} 3 \\ -1 \end{pmatrix}$. Thus

$$A^T A = \begin{pmatrix} 3 & 3 \\ 3 & 11 \end{pmatrix} = \frac{1}{\sqrt{10}} \begin{pmatrix} 1 & 3 \\ 3 & -1 \end{pmatrix} \begin{pmatrix} 12 & 0 \\ 0 & 2 \end{pmatrix} \frac{1}{\sqrt{10}} \begin{pmatrix} 1 & 3 \\ 3 & -1 \end{pmatrix}^T$$

2. (3 points) Let A be the matrix given in question 1. For what x does $x^T A^T A x$ reach its maximum? What is the maximum value of $x^T A^T A x$?

$x = \frac{1}{\sqrt{10}} \begin{pmatrix} 1 \\ 3 \end{pmatrix}$, where the maximum value of $x^T A^T A x = 12$

3. (3 points) Show that if $v \in \text{Null}(A^T A)$, then $v \in \text{Null}(A)$.

$$v \in \text{Null}(A^T A) \iff A^T A v = 0 \implies v^T A^T A v = 0 \iff (Av) \cdot (Av) = 0 \iff Av = 0$$