Name: ______________________ Section: __________

Math 54 Lec 006 Quiz 8

Friday, July 20, 2018

Justify your assertions; include detailed explanation, and show your work. Closed book exam, no sheet of notes and no calculator. This quiz is worth 9 points total.

1. (3 points) Let $A = \begin{pmatrix} 1 & 2 \\ -1 & 4 \\ 1 & 2 \end{pmatrix}$ and $b = \begin{pmatrix} 3 \\ -1 \\ 5 \end{pmatrix}$. Find the lease square solution to $Ax = b$.

   $A^T A = \begin{pmatrix} 3 & 0 \\ 0 & 24 \end{pmatrix}$ and $A^T b = \begin{pmatrix} 9 \\ 12 \end{pmatrix}$, so the lease square solution to $Ax = b$, given by solving $A^T Ax = A^T b$, is $\begin{pmatrix} 3 \\ \frac{1}{2} \end{pmatrix}$

2. (3 points) Let $V = P_2$ and pick three numbers $-2, 0, 2$. Define the inner product on $P_2[t]$ by $\langle p(t), q(t)\rangle = p(-2)q(-2) + p(0)q(0) + p(2)q(2)$. Show that $\{x^2 - 2x, x^2 + 2x, x^2 - 4\}$ is an orthogonal basis of P_2.

$$\langle x^2 - 2x, x^2 + 2x, \rangle = 8 * 0 + 0 * 0 + 0 * 8 = 0$$
$$\langle x^2 - 2x, x^2 - 4\rangle = 8 * 0 + 0 * (-4) + 0 * 8 = 0$$
$$\langle x^2 + 2x, x^2 - 4\rangle = 0 * 0 + 0 * (-4) + 8 * 0 = 0$$

3. (3 points) Orthogonally diagonalize the matrix $\begin{pmatrix} 6 & -2 \\ -2 & 9 \end{pmatrix}$

The eigenvalues are given by solving $(6-x)(9-x)-4 = x^2-15x+50$ which has roots $\lambda_1 = 10, \lambda_2 = 5$. For $\lambda_1 = 10$, we have eigenvector $\frac{1}{\sqrt{5}}\begin{pmatrix} 1 \\ -2 \end{pmatrix}$, while for $\lambda_2 = 5$, we have eigenvector $\frac{1}{\sqrt{5}}\begin{pmatrix} 2 \\ 1 \end{pmatrix}$. Thus $\begin{pmatrix} 6 & -2 \\ -2 & 9 \end{pmatrix} = \frac{1}{\sqrt{5}}\begin{pmatrix} 1 & 2 \\ -2 & 1 \end{pmatrix}\begin{pmatrix} 10 & 0 \\ 0 & 5 \end{pmatrix}\frac{1}{\sqrt{5}}\begin{pmatrix} 1 & 2 \\ -2 & 1 \end{pmatrix}^T$