

Name: \_\_\_\_\_

Section: \_\_\_\_\_

**Math 54 Lec 006 Quiz 7**

Tuesday, July 17, 2018

Justify your assertions; include detailed explanation, and show your work. Closed book exam, no sheet of notes and no calculator. This quiz is worth 9 points total.

1. (3 points) Let  $W = \text{Span} \left\{ \begin{pmatrix} 0 \\ 0 \\ 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 2 \\ -2 \\ 5 \\ -4 \end{pmatrix}, \begin{pmatrix} 2 \\ -2 \\ 0 \\ 0 \end{pmatrix} \right\}$ . Use Gram-Schmidt to find an orthogonal

basis for  $W$ , and then find  $\text{Proj}_W \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}$

Let  $v_1 = \begin{pmatrix} 0 \\ 0 \\ 1 \\ 1 \end{pmatrix}$ ,  $v_2 = \begin{pmatrix} 2 \\ -2 \\ 0 \\ 0 \end{pmatrix}$ , and  $v_3 = \begin{pmatrix} 2 \\ -2 \\ 5 \\ -4 \end{pmatrix}$ . We have

$$w_1 = v_1$$

$$w_2 = v_2 - \text{Proj}_{w_1} v_2 = v_2$$

$$w_3 = v_3 - \text{Proj}_{w_1} v_3 - \text{Proj}_{w_2} v_3 = \begin{pmatrix} 2 \\ -2 \\ 5 \\ -4 \end{pmatrix} - \frac{1}{2} \begin{pmatrix} 0 \\ 0 \\ 1 \\ 1 \end{pmatrix} - \begin{pmatrix} 2 \\ -2 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ \frac{9}{2} \\ -4 \end{pmatrix}$$

2. (3 points) Let  $W$  be the same vector subspace of  $\mathbb{R}^4$  in question 1. Find  $\text{Proj}_W \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}$

$$\text{Proj}_W e_1 = 0 \begin{pmatrix} 0 \\ 0 \\ 1 \\ 1 \end{pmatrix} + \frac{2}{8} \begin{pmatrix} 2 \\ -2 \\ 0 \\ 0 \end{pmatrix} + 0 \begin{pmatrix} 0 \\ 0 \\ \frac{9}{2} \\ \frac{9}{2} \end{pmatrix} = \begin{pmatrix} \frac{1}{2} \\ -\frac{1}{2} \\ 0 \\ 0 \end{pmatrix}$$

3. (3 points) True or False: If  $A, B$  are both  $n \times n$  orthogonal matrices, then so is  $AB$ .

True. We compute  $(AB)^T(AB)$ .

$$(AB)^T(AB) = B^T A^T AB = B^T IB = B^T B = I$$

so we see that  $AB$  is orthogonal.