Name: ________________________________ Section: ______________

Math 54 Lec 006 Quiz 6

Friday, July 13, 2018

Justify your assertions; include detailed explanation, and show your work. Closed book exam, no sheet of notes and no calculator. This quiz is worth 9 points total.

1. (3 points) Let

$$A = \begin{pmatrix} -3 & 0 & 4 \\ 0 & -1 & 0 \\ -2 & 7 & 3 \end{pmatrix}$$

Find the characteristic polynomial, and find all real eigenvalues with corresponding multiplicities.

$$det(A - \lambda I) = det \begin{pmatrix} -3-\lambda & 0 & 4 \\ 0 & -1-\lambda & 0 \\ -2 & 7 & 3-\lambda \end{pmatrix} = (-1-\lambda)((-3-\lambda)(3-\lambda)+8) = -(\lambda+1)^2(\lambda-1)$$

So the eigenvalues are -1 and 1 with multiplicities 2 and 1 respectively.

2. (3 points) True of False: If A is diagonalizable and invertible, then A^{-1} is also diagonalizable.

True. Since A is invertible, the diagonals of D are nonzero (denote the diagonal by $\lambda_1, ..., \lambda_n$), and hence D is invertible with inverse D^{-1} consisting of diagonal $\frac{1}{\lambda_1}, ..., \frac{1}{\lambda_n}$. Now

$$A^{-1} = (PDP^{-1})^{-1} = PD^{-1}P$$

So A^{-1} is diagonalizable.

3. (3 points) Determine if the matrix

$$\begin{pmatrix} 1 & 1 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

is diagonalizable. If so, diagonalize it. If not, explain why.

The characteristic polynomial is $(1-\lambda)\lambda^2$, and so the eigenvalues are 1 and 0. For $\lambda = 1$, the eigenvector is $\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$. For $\lambda = 0$, the eigenvectors are $\begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix}, \begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix}$. Thus we see that the matrix is diagonalizable with

$$P = \begin{pmatrix} 1 & -1 & -1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, D = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$