Name: ________________________________ Section: ______________

Math 54 Lec 006 Quiz 5

Friday, July 06, 2018

Justify your assertions; include detailed explanation, and show your work. Closed book exam, no sheet of notes and no calculator. This quiz is worth 9 points total.

1. (3 points) Let

$$H = \left\{ \begin{pmatrix} a \\ b \\ c \\ d \end{pmatrix} : a - 2b + c = 0, 2b + 3c = 0, a + b + 4c = 0 \right\}$$

be a subspace of $\mathbb{R}^4$. Find a basis for H and state the dimension of H.

The set H can be written as

$$H = \left\{ \begin{pmatrix} a \\ b \\ c \\ d \end{pmatrix} : \begin{pmatrix} 1 & -2 & 1 & 0 \\ 0 & 2 & 3 & 0 \\ 1 & 1 & 4 & 0 \end{pmatrix} \begin{pmatrix} a \\ b \\ c \\ d \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \right\} = \text{Null} \begin{pmatrix} 1 & -2 & 1 & 0 \\ 0 & 2 & 3 & 0 \\ 1 & 1 & 4 & 0 \end{pmatrix}$$

so finding a basis for H is the same thing as finding the Null of the above matrix.

$$\begin{pmatrix} 1 & -2 & 1 & 0 \\ 0 & 2 & 3 & 0 \\ 1 & 1 & 4 & 0 \end{pmatrix} \to \begin{pmatrix} 1 & -2 & 1 & 0 \\ 0 & 2 & 3 & 0 \\ 0 & 3 & 3 & 0 \end{pmatrix} \to \begin{pmatrix} 1 & -2 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 2 & 3 & 0 \end{pmatrix} \to \begin{pmatrix} 1 & -2 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix} \to \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

d is free, and $a = b = c = 0$, so a basis of H is $\begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}$ which is 1-dimensional.

2. (3 points) Let $B = \{7+5x, -3-x\}$ and $C = \{1-5x, -2+2x\}$ be two basis for $P_1[x]$, find the change-of-coordinates matrix from B to C.

$$\left(\begin{array}{cc|cc} 1 & -2 & 7 & -3 \\ -5 & 2 & 5 & -1 \end{array}\right) \rightarrow \left(\begin{array}{cc|cc} 1 & 0 & -3 & 1 \\ 0 & 1 & -5 & 2 \end{array}\right)$$

the change of coordinate matrix is $\begin{pmatrix} -3 & 1 \\ -5 & 2 \end{pmatrix}$

3. (3 points) True of False: If T is a linear transformation from V to $\mathbb{R}^4$, and if $ker(T) = \{0\}$, then $dim(V) \leq 4$.

The rank theorem for T states that

$$\dim(\operatorname{Im}(T)) + \dim(\operatorname{Ker}(T)) = \dim(V)$$

We were given that $\dim(\operatorname{Ker}(T)) = 0$. Now $\operatorname{Im}(T) \subset \mathbb{R}^4$ is a subspace, we see that $\dim(\operatorname{Im}(T)) \leq \dim(\mathbb{R}^4) = 4$, so putting all of these information in the rank theorem of T we see that $\dim(V) \leq 4$.