Name: ________________________________ Section: _____________

Math 54 Lec 006 Quiz 4

Friday, July 3, 2018

Justify your assertions; include detailed explanation, and show your work. Closed book exam, no sheet of notes and no calculator. This quiz is worth 9 points total.

1. (3 points) Let

$$A = \begin{pmatrix} 1 & 6 & 2 & -4 \\ -3 & 2 & -2 & -8 \\ 4 & -1 & 3 & 9 \end{pmatrix}$$

Find a basis for the column space.

$$\begin{pmatrix} 1 & 6 & 2 & -4 \\ -3 & 2 & -2 & -8 \\ 4 & -1 & 3 & 9 \end{pmatrix} \to \begin{pmatrix} 1 & 6 & 2 & -4 \\ 0 & 20 & 4 & -20 \\ 0 & -25 & -5 & 25 \end{pmatrix} \to \begin{pmatrix} 1 & 6 & 2 & -4 \\ 0 & 20 & 4 & -20 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

so a basis for the column space is

$$\left\{ \begin{pmatrix} 1 \\ -3 \\ 4 \end{pmatrix}, \begin{pmatrix} 6 \\ 2 \\ -1 \end{pmatrix} \right\}$$

2. (3 points) Let $T : P_2[t] \to \mathbb{R}$ be the linear transformation defined by $T(p) = p(1)$. Find the polynomial p in $P_2[t]$ that spans the kernel of T, and describe the image of T.

The kernel of T is $\{p(t) : p(1) = 0\}$ so it consists of polynomials where 1 is a root. The set can be described by $\{p(t) = at^2 + bt + c : p(1) = a + b + c = 0\}$ so it consists of polynomials where the coefficient adds to zero.

Alternatively, it can be described by polynomials having 1 as a root. So in P_2 the polynomials that has 1 as a root can be spanned by $(x - 1)$ and $x^2 - 1$.

The image of T is the entire $\mathbb{R}$ since $p(1) = a + b + c$ can be any real number.

3. (3 points) True of False: Suppose H and K are subspaces of V, then $H \cup K$ is also a subspace of V.

False. Let $V = \mathbb{R}^2$, and let $H = span\left\{\begin{pmatrix} 1 \\ 0 \end{pmatrix}\right\}$ and $K = span\left\{\begin{pmatrix} 0 \\ 1 \end{pmatrix}\right\}$, then we see that $\begin{pmatrix} 1 \\ 0 \end{pmatrix} \in H \cup K$ and $\begin{pmatrix} 0 \\ 1 \end{pmatrix} \in H \cup K$ but their sum $\begin{pmatrix} 1 \\ 1 \end{pmatrix} \notin H \cup K$.