Name: ________________________ Section: __________

Math 54 Lec 006 Quiz 1

Friday, June 22, 2018

Justify your assertions; include detailed explanation, and show your work. Closed book exam, no sheet of notes and no calculator. This quiz is worth 9 points total.

1. (3 points) Let

$$A = \begin{pmatrix} 1 & -2 & 4 & 0 \\ 3 & -7 & 10 & 2 \\ 2 & -5 & 6 & 2 \end{pmatrix}$$

(a) Describe the Null space of A. Is $\begin{pmatrix} 4 \\ 4 \\ 1 \\ 3 \end{pmatrix}$ in the Null space of A?

$$A = \begin{pmatrix} 1 & -2 & 4 & 0 \\ 3 & -7 & 10 & 2 \\ 2 & -5 & 6 & 2 \end{pmatrix} \to \begin{pmatrix} 1 & -2 & 4 & 0 \\ 0 & -1 & -2 & 2 \\ 0 & -1 & -2 & 2 \end{pmatrix} \to \begin{pmatrix} 1 & -2 & 4 & 0 \\ 0 & -1 & -2 & 2 \\ 0 & 0 & 0 & 0 \end{pmatrix} \to \begin{pmatrix} 1 & 0 & 8 & -4 \\ 0 & 1 & 2 & -2 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

So the Null space is

$$\text{Span}\left\{ \begin{pmatrix} -8 \\ -2 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 4 \\ 2 \\ 0 \\ 1 \end{pmatrix} \right\}$$

Finally, since

$$\begin{pmatrix} 1 & -2 & 4 & 0 \\ 3 & -7 & 10 & 2 \\ 2 & -5 & 6 & 2 \end{pmatrix} \begin{pmatrix} 4 \\ 4 \\ 1 \\ 3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

the vector $\begin{pmatrix} 4 \\ 4 \\ 1 \\ 3 \end{pmatrix}$ is in the nullspace of A.

(b) Does $Ax = b$ have a solution for all $b \in \mathbb{R}^3$?

No. The third row does not have a pivot.

2. (3 points) Let

$$A = \begin{pmatrix} 1 & 1 & 2 \\ 1 & 0 & 3 \\ 3 & x & 1 \end{pmatrix}$$

(a) For what x are the columns of A linearly independent?

(b) For what x do the columns of A span $\mathbb{R}^3$?

$$\begin{pmatrix} 1 & 1 & 2 \\ 1 & 0 & 3 \\ 3 & x & 1 \end{pmatrix} \to \begin{pmatrix} 1 & 1 & 2 \\ 0 & -1 & 1 \\ 0 & x-3 & -5 \end{pmatrix} \to \begin{pmatrix} 1 & 1 & 2 \\ 0 & -1 & 1 \\ 0 & 0 & x-8 \end{pmatrix}$$

So in order for it to have a pivot in every row (or every column), $x \neq 8$.

3. (3 points) True of False: Suppose $\{v_1, v_2, v_3\}$ is linearly independent, then so is $\{v_1 + v_2, v_1 - v_2, v_3\}$.

True. Consider a linear combination $a(v_1+v_2)+b(v_1-v_2)+cv_3 = 0$, we want to show that $a = b = c = 0$. Expanding the expression, we have $(a + b)v_1 + (a - b)v_2 + cv_3 = 0$, and since $\{v_1, v_2, v_3\}$ is linearly independent, $(a + b) = (a - b) = c = 0$, so $a = b = c = 0$. This shows that $\{v_1 + v_2, v_1 - v_2, v_3\}$ is linearly independent.