Math 54 Section 701 Handout 9

July 6, 2018

Question 1.

$$\left(\begin{array}{cc|cc} 7 & -3 & 1 & -2 \\ 5 & -1 & -5 & 2 \end{array}\right) \to \left(\begin{array}{cc|cc} 1 & 0 & -2 & 1 \\ 0 & 1 & -5 & 3 \end{array}\right)$$

Question 2.

$$\left(\begin{array}{cc|cc} -1 & 1 & 1 & 1 \\ 8 & -5 & 4 & 1 \end{array}\right) \to \left(\begin{array}{cc|cc} 1 & 0 & 3 & 2 \\ 0 & 1 & 4 & 3 \end{array}\right)$$

Question 3.

Since

$$\begin{pmatrix} 3 & 7 & 9 \\ -4 & -5 & 1 \\ 2 & 4 & 4 \end{pmatrix} \begin{pmatrix} 4 \\ -3 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} = 0 \begin{pmatrix} 4 \\ -3 \\ 1 \end{pmatrix}$$

we see that $\begin{pmatrix} 4 \\ -3 \\ 1 \end{pmatrix}$ is an eigenvector with eigenvalue 0.

Question 4.

The eigenspace is just the null of $A - 3I$, which is

$$\begin{pmatrix} 4-3 & 2 & 3 \\ -1 & 1-3 & -3 \\ 2 & 4 & 9-3 \end{pmatrix} = \begin{pmatrix} 1 & 2 & 3 \\ -1 & -2 & -3 \\ 2 & 4 & 6 \end{pmatrix} \to \begin{pmatrix} 1 & 2 & 3 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

so the eigenspace is spanned by two vectors

$$\left\{ \begin{pmatrix} -2 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} -3 \\ 0 \\ 1 \end{pmatrix} \right\}$$

Question 5.

No. Since T is rotation by $\theta \in (0, \pi)$, it cannot send any vector to some multiple of itself.

Yes. For example, the vector e_2 is an eigenvector with eigenvalue 1.

Question 6.

True of False:

1. False.

$$\begin{pmatrix} 2 & 0 \\ 0 & 1 \end{pmatrix} \to \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

   changes the eigenvalue from $\lambda = 1, 2$ to $\lambda = 1$.

2. False. The 2×2 identity matrix has e_1 and e_2 as eigenvectors both with eigenvalue 1, but they are linearly independent.