# Math 54 Handout 8

July 2, 2018

### Question 1.

Find a basis for $Null(A)$, where

$$A = \begin{pmatrix} 1 & 5 & -4 & -3 & 1 \\ 0 & 1 & -2 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

### Question 2.

Find a basis for the subspace spanned by the given vectors:

$$\left\{ \begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix}, \begin{pmatrix} 3 \\ 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 9 \\ 4 \\ -2 \end{pmatrix}, \begin{pmatrix} -7 \\ -3 \\ 1 \end{pmatrix} \right\}$$

### Question 3.

The first four Laguerre polynomials are $1, 1-t, 2-4t+t^2, 6-18t+19t^2-t^3$. Show that these polynomials form a basis of $P_3[t]$. Compute the coordinates of $7-8t+3t^2$ in this basis.

### Question 4.

True of False: Given $V$, $W$ subspaces of some vector space $H$, then their intersection $U \bigcap V$ is also a subspace of $H$.

### Question 5.

Let $M_{2\times 2}$ be the vector space of all $2 \times 2$ matrices, and define $T : M_{2\times 2} \to M_{2\times 2}$ by $T(A) = A + A^T$.

1. Show that $T$ is a linear transformation.
2. Let $B$ be in $M_{2\times 2}$ such that $B = B^T$, find an $A$ such that $T(A) = B$.
3. Show that the range of $T$ is the set of $B$ in $M_{2\times 2}$ with the property that $B^T = B$.
4. Describe the kernel of $T$.