Math 54 Handout 7

July 3, 2018

Question 1.

False: $0 \notin S$.

Question 2.

1. $(T_1 + T_2)(\mathbf{v}) = T_1(\mathbf{v}) + T_2(\mathbf{v}) = T_2(\mathbf{v}) + T_1(\mathbf{v}) = (T_2 + T_1)(\mathbf{v})$ for all $\mathbf{v}$
2. $((T_1 + T_2) + T_3)(\mathbf{v}) = (T_1 + T_2)(\mathbf{v}) + T_3(\mathbf{v}) = T_1(\mathbf{v}) + T_2(\mathbf{v}) + T_3(\mathbf{v}) = (T_1 + (T_2 + T_3))(\mathbf{v})$ for all $\mathbf{v}$
3. The zero function E that sends $\mathbf{v} \mapsto 0$ for all $\mathbf{v}$ is the zero vector in this vector space since $(T+E)(\mathbf{v}) = T(\mathbf{v}) + E(\mathbf{v}) = T(\mathbf{v}) + 0 = T(\mathbf{v})$ for all $\mathbf{v}$
4. Given T, the function S that sends $\mathbf{v} \mapsto -T(\mathbf{v})$ satisfy $(T+S)(\mathbf{v}) = T(\mathbf{v})+S(\mathbf{v}) = T(\mathbf{v})+(-T(\mathbf{v})) = 0$ for all $\mathbf{v}$, thus $T + S = E$.
5. $(c(T_1 + T_2))(\mathbf{v}) = c(T_1(\mathbf{v}) + T_2(\mathbf{v})) = cT_1(\mathbf{v}) + cT_2(\mathbf{v}) = (cT_1 + cT_2)(\mathbf{v})$ for all $\mathbf{v}$
6. $((c + d)T)(\mathbf{v}) = (c + d)(T(\mathbf{v})) = cT(\mathbf{v}) + dT(\mathbf{v}) = (cT + dT)(\mathbf{v})$ for all $\mathbf{v}$
7. $(c(dT))(\mathbf{v}) = c((dT)(\mathbf{v})) = cdT(\mathbf{v}) = (cd)T(\mathbf{v}) = ((cd)T)(\mathbf{v})$ for all $\mathbf{v}$
8. $(1T)(\mathbf{v}) = 1(T(\mathbf{v})) = T(\mathbf{v})$ for all $\mathbf{v}$