Math 54 Handout 6

June 28, 2018

Question 1.

$$\det\begin{pmatrix}1 & 1 & 2\\ 1 & 0 & 3\\ 3 & 8 & 1\end{pmatrix} = (-1)\det\begin{pmatrix}1 & 2\\ 8 & 1\end{pmatrix} + (-3)\det\begin{pmatrix}1 & 1\\ 3 & 8\end{pmatrix} = 15 - 15 = 0$$

Question 2.

Since $AA^{-1} = I$, by taking determinants on both sides we get

$$\det(AA^{-1}) = det(I) = 1$$

since $\det(AA^{-1}) = \det(A)\det(A^{-1})$ we see that $\det(A)\det(A^{-1}) = 1$, proving that

$$det(A^{-1}) = \frac{1}{det(A)}$$

Question 3.

False. Counterexample:

$$A = \begin{pmatrix}1 & 0\\ 0 & 0\end{pmatrix}, B = \begin{pmatrix}0 & 0\\ 0 & 1\end{pmatrix}$$

Question 4.

False.

$$\det\left(\begin{pmatrix}0 & 2\\ 1 & 0\end{pmatrix}\begin{pmatrix}0 & 3\\ 0 & 0\end{pmatrix} - \begin{pmatrix}0 & 3\\ 0 & 0\end{pmatrix}\begin{pmatrix}0 & 2\\ 1 & 0\end{pmatrix}\right) = \det\begin{pmatrix}-3 & 0\\ 0 & 3\end{pmatrix} = -9 \neq 0$$

Question 5.

$1 = det(I) = det(UU^T) = det(U)det(U^T) = det(U)det(U) = (det(U))^2$, so $det(U) = \pm 1$.

Question 6.

1. First of all,

$$S = \left\{ \begin{pmatrix} x \\ y \\ z \end{pmatrix} : x_1^2 + x_2^2 + x_3^2 \leq 1 \right\}$$

   Since

$$\begin{pmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} ax \\ by \\ cz \end{pmatrix}$$

   we let $x' = ax, y' = by, z' = cz$ be the coordinates for the codomain, then the set

$$T(S) = \left\{ \begin{pmatrix} x' \\ y' \\ z' \end{pmatrix} : \frac{x_1^2}{a} + \frac{x_2^2}{b} + \frac{x_3^2}{c} \leq 1 \right\}$$

2. The volume of the unit ball is $\frac{4}{3}\pi$, so the volume of the ellipsoid is $\frac{4}{3}\pi det(T) = \frac{4}{3}\pi abc$.

Question 7.

$$x_1 = \frac{det(A_1(b))}{det(A)} = \frac{5}{6}, x_2 = \frac{det(A_2(b))}{det(A)} = \frac{-1}{6}$$