

Math 54 Handout 6

June 28, 2018

Question 1.

Let

$$A = \begin{pmatrix} 1 & 1 & 2 \\ 1 & 0 & 3 \\ 3 & 8 & 1 \end{pmatrix}$$

Compute the determinant of A .

Question 2.

Show that if A is invertible, then

$$\det(A^{-1}) = \frac{1}{\det(A)}$$

Question 3.

True or False:

$$\det(A + B) = \det(A) + \det(B)$$

Question 4.

True or False: Assume A, B are square matrices, then $\det(AB - BA) = 0$.

Question 5.

Suppose $U^T U = I$ (such matrices are called orthogonal matrices). Show that $\det(U) = \pm 1$.

Question 6.

Let $T : \mathbb{R}^3 \rightarrow \mathbb{R}^3$ be the linear transformation described by the matrix

$$\begin{pmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{pmatrix}$$

where a, b, c are positive integers. Let S be the unit ball described by the equation $x_1^2 + x_2^2 + x_3^2 \leq 1$.

1. Show that $T(S)$ is an ellipsoid described by the equation

$$\frac{x_1^2}{a} + \frac{x_2^2}{b} + \frac{x_3^2}{c} \leq 1$$

2. Calculate the volume of the ellipsoid $T(S)$.

Question 7.

Use Cramer's rule to solve the system

$$\begin{cases} 5x_1 + 7x_2 &= 3 \\ 2x_1 + 4x_2 &= 1 \end{cases}$$