Math 54 Handout 5

June 25, 2018

Question 1.

Suppose x is a solution to $Ax = 0$. Multiplying by C on both sides of the equation $Ax = 0$, we see that $x = Ix = CAx = C0 = 0$, and so $Ax = 0$ has only the trivial solution.

Question 2.

We want to solve for $Ax = b$ for any given b. Multiplying by b on both sides of the equation $AD = I$, we obtain $ADb = Ib = b$, and so $x = Db$ is a solution to $Ax = b$ for any given b.

Question 3.

If AB is invertible, then there exists M such that $(AB)M = I$, but $(AB)M = A(BM) = I$, so A is invertible with inverse BM. Similarly, since $M(AB) = I$, we see that $(MA)B = I$, and hence B is invertible with inverse MA.

Question 4.

$$\begin{pmatrix} 1 & 1 & 2 \\ 1 & 0 & 3 \\ 3 & 8 & 1 \end{pmatrix} \to \begin{pmatrix} 1 & 1 & 2 \\ 0 & -1 & 1 \\ 0 & 5 & -5 \end{pmatrix} \to \begin{pmatrix} 1 & 1 & 2 \\ 0 & -1 & 1 \\ 0 & 0 & 0 \end{pmatrix}$$

Since there is not a pivot in every row and column, this matrix is not invertible.

Question 5.

$$\left(\begin{array}{cc|cc} 3 & 7 & 1 & 0 \\ 2 & 5 & 0 & 1 \end{array}\right) \to \left(\begin{array}{cc|cc} 3 & 7 & 1 & 0 \\ 0 & \frac{1}{3} & -\frac{2}{3} & 1 \end{array}\right) \to \left(\begin{array}{cc|cc} 3 & 7 & 1 & 0 \\ 0 & 1 & -2 & 3 \end{array}\right)$$

$$\to \left(\begin{array}{cc|cc} 3 & 0 & 15 & -21 \\ 0 & 1 & -2 & 3 \end{array}\right) \to \left(\begin{array}{cc|cc} 1 & 0 & 5 & -7 \\ 0 & 1 & -2 & 3 \end{array}\right)$$

Question 6.

False. Counterexample:

$$A = \begin{pmatrix} 2 \end{pmatrix}, B = \begin{pmatrix} 3 \end{pmatrix}$$